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Complete resummation of chirally-enhanced loop-effects in the MSSM with non-minimal sources of flavor-violation

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Abstract

In this article we present the complete resummation of the leading chirally-enhanced corrections stemming from gluino-squark, chargino-sfermion and neutralino-sfermion loops in the MSSM with non-minimal sources of flavor-violation. We compute the finite renormalization of fermion masses and the CKM matrix induced by chirality-flipping self-energies. In the decoupling limit $M_{\rm SUSY} \gg v$, which is an excellent approximation to the full theory, we give analytic results for the effective gaugino(higgsino)-fermion-sfermion and the Higgs-fermion-fermion vertices. Using these vertices as effective Feynman rules, all leading chirally-enhanced corrections can consistently be included into perturbative calculations of Feynman amplitudes. We also give a generalized parametrization for the bare CKM matrix which extends the classic Wolfenstein parametrization to the case of complex parameters λ and A.

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I. INTRODUCTION

In the Standard model (SM) left- and right-handed fermion fields f_L and f_R transform differently under the $SU(2)_L$ gauge symmetry. Thus, the requirement of gauge invariance forbids explicit mass-terms. Instead these fields acquire masses via the Higgs mechanism. The Higgs field H (which is itself a $SU(2)_L$ doublet) couples left-handed fermions to right-handed ones with coupling strength Y^{f_i} (i denotes the generation of the fermion), so that the non-vanishing vacuum expectation value[1] (vev) v of H then induces fermion masses $m_{f_i} = Y^{f_i} v$. Experimental measurements revealed $m_{f_i} \ll v$ for all the fermions except for the top quark implying $Y^{f_i} \ll 1$ ($f_i \neq t$). Since the Yukawa couplings Y^{f_i} ($f_i \neq t$) are thus small compared to the gauge couplings, their values are in principle sensitive to loop corrections if such higher-order contributions manage to escape the Y^{f_i}-suppression. However, any loop correction to Y^{f_i} has to involve a chirality-flip and since in the SM the Yukawa couplings are the only sources of chirality-violation the loop must be proportional to Y^{f_i} itself, so that the Y^{f_i}-suppression cannot be avoided.

In the Minimal Supersymmetric Standard Model (MSSM) the situation is different. Firstly, it contains two Higgs doublets H_u and H_d coupling to up- and down-type quark (lepton) superfields, respectively. The neutral components of these Higgs fields acquire vevs v_u and v_d with $v_u^2 + v_d^2 = v^2$. If there is a hierarchy $v_d \ll v_u$, one faces enhanced corrections to Feynman amplitudes in which the tree-level contribution is suppressed by the small vev v_d while the loop correction involves v_u instead. In this case the ratio of one-loop to tree-level contribution receives an enhancement factor $\tan\beta \equiv v_u/v_d$ [1]. Secondly, the MSSM offers another source of chirality-flips, namely the soft SUSY-breaking trilinear Higgs-sfermion couplings A^f (A-terms) with mass dimension one[2]. Whereas one has $A^f \propto Y^f$ in a scenario of Minimal Flavor Violation (MFV) [2], in the general MSSM these couplings are independent free parameters. Thus, enhanced corrections to Feynman amplitudes in which the tree-level contribution is suppressed by a small Y^f while the loop correction involves A^f instead are possible. In such a case the ratio of the one-loop to the tree-level contribution receives an enhancement factor $A^f_{ij}/(Y^f_{ij} M_{\rm SUSY})$, where $M_{\rm SUSY}$ is a typical SUSY-mass.

In both cases the respective enhancement factor ($\tan\beta$ or $A^f_{ij}/(Y^f_{ij} M_{\rm SUSY})$) can compensate for the loop suppression. Therefore such a higher loop correction can be of the same size, or even larger, as the leading order diagram[3] and perturbative calculations (using the usual counting in powers of α_s, $\alpha_{1,2}$) should thus be supplemented by an all-order resummation of the enhanced corrections. In nearly all cases this can be achieved by using effective Feynman rules which incorporate the resummed corrections. Such effective rules have already been calculated in the literature for several special cases and vertices ($S^0 = H^0, A^0, h^0$):

[1] We define $\langle H \rangle = v$ (without a factor $\sqrt{2}$), so that $v \approx 174\,\text{GeV}$.

[2] Strictly speaking, the flip of fermion chirality is provided by a gaugino propagator in the corresponding loop diagram. However, the A-terms change the $SU(2)_L$-charge on the sfermion-line and in this sense they are also necessary in order to mediate the chirality-flip of the fermions.

[3] Since self-energy diagrams involving A^f-terms can be of the same order as the light fermion masses, they can even generate them entirely in a scenario with loop-induced soft Yukawa couplings [3, 4, 5, 6]

- $S^0 b\bar{b}$ and $H^+ t\bar{b}$ vertices for $A^b = 0$ [1, 7].
- $S^0 d_i \bar{d}_j$ and $H^+ u_i \bar{d}_j$ vertices in the MSSM with MFV [8, 9, 10, 11, 12, 13].
- $S^0 d_i \bar{d}_j$ and $H^+ u_i \bar{d}_j$ vertices in the MSSM with MFV and additional sources of CP violation [14, 15, 16][4].
- $S^0 f_i \bar{f}_j$ and $H^+ f_i \bar{f}'_j$ vertices for quarks and leptons in the general MSSM in the limit $A^f = 0$ [17, 18].
- $S^0 b\bar{b}$ vertex for $A^b \neq 0$ including NNLO QCD corrections [19].
- $\tilde{g}\tilde{d}_i\bar{d}_j$, $\widetilde{\chi}^+\tilde{f}_i\bar{f}'_j$, $\widetilde{\chi}^0\tilde{f}_i\bar{f}_j$ vertices for quarks and leptons in the general MSSM for $A^f = 0$ [20]. This method requires iterative resummation.
- Complete set of $S^0 f_i \bar{f}_j$, $H^+ f_i \bar{f}'_j$, $\tilde{g}\tilde{d}_i\bar{d}_j$, $\widetilde{\chi}^+\tilde{u}_i\bar{d}_j$, $\widetilde{\chi}^-\tilde{d}_i\bar{u}_j$, $\widetilde{\chi}^0\tilde{d}_i\bar{d}_j$ vertices in the MSSM with MFV beyond the decoupling limit $M_{\rm SUSY} \gg v$ [21].
- Effective $\widetilde{\chi}^+\tilde{\ell}_i\bar{\nu}_j$ and $\widetilde{\chi}^0\tilde{\ell}_i\bar{\ell}_j$, vertices in the general MSSM (with $A^\ell = 0$) beyond the decoupling limit $M_{\rm SUSY} \gg v$ [22].
- $\tilde{g}\tilde{d}_i\bar{d}_j$, $\widetilde{\chi}^+\tilde{u}_i\bar{d}_j$, $\widetilde{\chi}^-\tilde{d}_i\bar{u}_j$ vertices in the general MSSM beyond the decoupling limit $M_{\rm SUSY} \gg v$ (corrections from gluino-squark loops only) [23, 24].
- $S^0 d_i \bar{d}_j$ and $H^+ u_i \bar{d}_j$ vertices in the general MSSM including A-terms, A' terms beyond leading order in $v/M_{\rm SUSY}$ (corrections from gluino-squark loops only) [25].

However, a complete list of the gaugino(higgsino)-fermion-sfermion and Higgs-fermion-fermion vertices including the full set of chirally enhanced corrections is still missing. In this article we deliver the missing pieces taking into account enhanced contributions from gluino-squark, chargino-sfermion and neutralino-sfermion loops in the general MSSM. For the resummation we rely on the methods developed in Refs. [7, 21, 23, 24], which can be applied for an arbitrary value of the SUSY mass scale $M_{\rm SUSY}$, in particular beyond the decoupling limit $M_{\rm SUSY} \gg v$. In general, however, the resummation of self-energy corrections requires iterative procedures and the enhanced vertex corrections to the Higgs-fermion-fermion vertex cannot be absorbed into an effective coupling. These complications do not occur if contributions which are subleading in $v/M_{\rm SUSY}$ are neglected. We present analytical resummation formulae in this limit, which for realistic values of SUSY masses turn out to be an excellent approximation to the full result: according to the new results of the CMS collaboration [26] and the Atlas experiment [27], squarks and gluinos must be rather heavy so that decoupling effects in squark-mixing can be neglected to a good approximation. Since $m_\tau < m_b$ and the off-diagonal A^ℓ-terms are severely constrained from experiments searching for flavor transitions in the charged lepton sector, the LR-elements in the slepton mass matrices are typically smaller than the ones in the squark mass matrices and

[4] Ref. [16] also extends the analysis to general soft-SUSY-breaking terms by expanding them in terms of the Yukawa couplings.

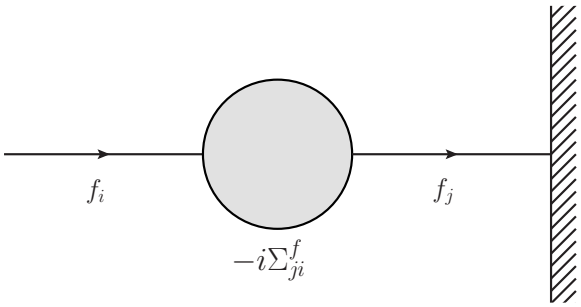

FIG. 1: Self-energy inducing wave-function rotation in flavor-space.

decoupling effects in slepton-mixing can only be important if the sleptons are much lighter than the squarks. Furthermore, chargino- and neutralino-mixing effects are suppressed by $M_W^2/M_{\rm SUSY}^2$ and can be neglected if only $M_{\rm SUSY} > M_W$. These facts support our statement that the decoupling limit is almost always an excellent approximation.

The paper is organized as follows. In Sec. II we calculate the chirally-enhanced parts of the quark and lepton self-energies in the MSSM. Sec. III is devoted to the renormalization of Yukawa couplings, fermion wave-functions and the CKM matrix in the presence of chirally-enhanced corrections. Our main result, the effective gaugino(higgsino)-fermion-sfermion and Higgs-fermion-fermion vertices are presented in Sec. IV A and Sec. IV B. We conclude in Sec. V. Our conventions and a generalization of the Wolfenstein parametrization to the case of complex λ and A parameters are given in the Appendix.

II. CHIRALLY-ENHANCED CONTRIBUTIONS TO SELF-ENERGIES

In this section we calculate all chirally-enhanced contributions from fermion self-energies in the general MSSM. We first give the complete formulae and then extract the leading order in $v/M_{\rm SUSY}$, up to which we will be able to give analytic results for the effective vertices.

A. General remarks

In general, it is possible to decompose any self-energy (see Fig. 1) into chirality-flipping and chirality-conserving parts in the following way (in what follows we denote the flavor of the incoming (outgoing or "final") fermion by i (j or f), respectively):

$$\Sigma^f_{ji}(p) = \left(\Sigma^{f\,LR}_{ji}(p^2) + \not{p}\Sigma^{f\,RR}_{ji}(p^2)\right) P_R + \left(\Sigma^{f\,RL}_{ji}(p^2) + \not{p}\Sigma^{f\,LL}_{ji}(p^2)\right) P_L \tag{1}$$

Note that the chirality-changing parts $\Sigma^{f\,LR}_{ji}$ and $\Sigma^{f\,RL}_{ji}$ have mass dimension 1 and are related through

$$\Sigma^{f\,LR}_{ji}(p^2) = \Sigma^{f\,RL*}_{ij}(p^2), \tag{2}$$

while the hermitian chirality-conserving parts $\Sigma^{f\,LL}_{ji} = \Sigma^{f\,LL\star}_{ij}$ and $\Sigma^{f\,RR}_{ji} = \Sigma^{f\,RR\star}_{ij}$ are dimensionless and in general not related to each other. Any loop contribution to the fermion self-energy involving sfermions and gluinos, charginos or neutralinos can be written as

$$\begin{aligned}
\Sigma^{f\tilde{\lambda}\,LR}_{ji}(p^2) &= \frac{-1}{16\pi^2}\sum_{s=1}^{6}\sum_{I=1}^{N} m_{\tilde{\lambda}_I}\Gamma^{\tilde{\lambda}_I L*}_{f_j\tilde{f}_s}\Gamma^{\tilde{\lambda}_I R}_{f_i\tilde{f}_s} B_0\left(p^2; m^2_{\tilde{\lambda}_I}, m^2_{\tilde{f}_s}\right), \\
\Sigma^{f\tilde{\lambda}\,RL}_{ji}(p^2) &= \frac{-1}{16\pi^2}\sum_{s=1}^{6}\sum_{I=1}^{N} m_{\tilde{\lambda}_I}\Gamma^{\tilde{\lambda}_I R*}_{f_j\tilde{f}_s}\Gamma^{\tilde{\lambda}_I L}_{f_i\tilde{f}_s} B_0\left(p^2; m^2_{\tilde{\lambda}_I}, m^2_{\tilde{f}_s}\right), \\
\Sigma^{f\tilde{\lambda}\,LL}_{ji}(p^2) &= \frac{-1}{16\pi^2}\sum_{s=1}^{6}\sum_{I=1}^{N} \Gamma^{\tilde{\lambda}_I L*}_{f_j\tilde{f}_s}\Gamma^{\tilde{\lambda}_I L}_{f_i\tilde{f}_s} B_1\left(p^2; m^2_{\tilde{\lambda}_k}, m^2_{\tilde{f}_s}\right), \\
\Sigma^{f\tilde{\lambda}\,RR}_{ji}(p^2) &= \frac{-1}{16\pi^2}\sum_{s=1}^{6}\sum_{I=1}^{N} \Gamma^{\tilde{\lambda}_I R*}_{f_j\tilde{f}_s}\Gamma^{\tilde{\lambda}_I R}_{f_i\tilde{f}_s} B_1\left(p^2; m^2_{\tilde{\lambda}_I}, m^2_{\tilde{f}_s}\right). \qquad (3)
\end{aligned}$$

Here $\tilde{\lambda}$ stands for the SUSY fermions $(\tilde{g}, \tilde{\chi^0}, \tilde{\chi^\pm})$ and N denotes their corresponding number (2 for charginos, 4 for neutralinos and 8 for gluinos). The coupling coefficients $\Gamma^{\tilde{\lambda}_I L(R)}_{f_i\tilde{f}_s}$ and the loop functions B_0 and B_1 are defined in Appendix D and in Appendix E. For low-energy decays with $p^2 \sim m_f^2 \ll M^2_{\rm SUSY}$, it is possible to expand the loop function in the small parameter $p^2/M^2_{\rm SUSY}$:

$$\begin{aligned}
B_0\left(p^2; m_1^2, m_2^2\right) &= B_0\left(m_1^2, m_2^2\right) + p^2 m_2^2 D_0\left(m_1^2, m_2^2, m_2^2, m_2^2\right) + \ldots \\
B_1\left(p^2; m_1^2, m_2^2\right) &= \frac{1}{2}C_2\left(m_1^2, m_2^2, m_2^2\right) + m^2 p^2 E_2\left(m_1^2, m_2^2, m_2^2, m_2^2, m_2^2\right) + \ldots \qquad (4)
\end{aligned}$$

For most processes, it is sufficient to evaluate the self-energies at vanishing external momentum. Further, only the chirality-flipping part of a self-energy $(\Sigma^{f\,LR}_{ji}, \Sigma^{f\,RL}_{ji})$ can be enhanced in the MSSM either by a factor $\tan\beta$ [1] or by a factor $A^f_{ij}/(Y^f_{ij} M_{\rm SUSY})$ [23]. Therefore, we neglect the chirality-conserving parts $\Sigma^{f\,LL,RR}_{ji}$ in the following.

We parametrize the 6×6 sfermion mixing matrices as

$$\mathcal{M}^2_f = \begin{pmatrix} \Delta^{f\,LL} & \Delta^{f\,LR} \\ \Delta^{f\,LR\dagger} & \Delta^{f\,RR} \end{pmatrix} \qquad (5)$$

with $\Delta^{f\,XY}$ being 3×3 matrices in flavor-space. The numerical values for the $\Delta^{f\,XY}_{ij}$ depend on the chosen basis for the sfermion fields. It is common to choose for the quark fields the basis in which the Yukawa couplings are dioagonal and, in order to have manifest supersymmetry in the superpotential, to subject the squarks to the same rotations as the quarks. The resulting basis for the super-fields is called super-CKM basis.

We choose the super-CKM basis for the squark mass matrices by requiring that the fundamental bare Yukawa couplings $Y^{q\,(0)}$ in the superpotential are diagonal in flavor space. As discussed in Ref. [24], such a definition of the super-CKM basis has several advantages compared to an "on-shell" definition in which the physical quark masses are diagonal instead:

- The definition of the $\Delta_{ij}^{q\,XY}$ does not depend on the renormalization scheme used for the fermion mass matrices m_{ij}^q.

- The basis for the $\Delta_{ij}^{q\,XY}$ is defined at the level of bare quantities, so that their definition remains valid to all orders in perturbation theory. A choice of basis with the renormalized Yukawa couplings Y^q being diagonal, on the other hand, requires a redefinition of the $\Delta_{ij}^{q\,XY}$ at every order in perturbation theory.

- In our super-CKM basis the squark mass matrices are diagonal in a scenario of flavor-blind SUSY breaking terms. If an on-shell definition is used instead, the bare Yukawa couplings $Y^{q(0)}$ entering the squark mass matrices are not diagonal anymore and the squark mass matrices develop flavor off-diagonal entries even in case of flavor-blind SUSY breaking terms.

The elements

$$\begin{aligned}
\Delta_{ij}^{u\,LR} &= -v_u A_{ij}^u - v_d A_{ij}^{\prime u} - v_d\,\mu\,Y^{u_i(0)}\,\delta_{ij}\,,\\
\Delta_{ij}^{d\,LR} &= -v_d A_{ij}^d - v_u A_{ij}^{\prime d} - v_u\,\mu\,Y^{d_i(0)}\,\delta_{ij}\,,\\
\Delta_{ij}^{\ell\,LR} &= -v_d A_{ij}^\ell - v_u A_{ij}^{\prime \ell} - v_u\,\mu\,Y^{\ell_i(0)}\,\delta_{ij}
\end{aligned} \tag{6}$$

and $\Delta_{ij}^{f\,RL} = \Delta_{ji}^{f\,LR*}$ flip the "chiralities". Appearing in gluino-squark, chargino-sfermion or neutralino-sfermion contributions to fermion self-energies, they generate chirality-enhanced effects with respect to the tree-level masses if they involve the large vev v_u ($\tan\beta$-enhancement for down-quark/lepton self-energies) or a trilinear $A^{(\prime)f}$-term ($A_{ij}^{(\prime)f}/(Y_{ij}^f M_{\rm SUSY})$-enhancement).

The couplings $\Gamma_{f_i\tilde{f}_s}^{\tilde{\lambda}_I L(R)}$ in Eq. (3) depend on the corresponding sfermion mixing matrix and thus on the elements $\Delta_{ij}^{f\,LR} \sim v M_{\rm SUSY}$ entering the sfermion mass matrices. As non-polynomial functions of these terms, the $\Gamma_{f_i\tilde{f}_s}^{\tilde{\lambda}_I L(R)}$ contain all orders in $(v/M_{\rm SUSY})^n$ ($n = 0, 1, 2, ...$). However, in the limit $M_{\rm SUSY} \gg v$ this power series rapidly converges and only the first terms in the expansion are relevant. The assumption $M_{\rm SUSY} \gg v$ is an excellent approximation to the full theory as soon as one takes into account bounds from direct SUSY searches [25]. Qualitatively this can be understood as follows: the off-diagonal mass-insertion terms induce a splitting of the sfermion masses of the form $m_{\tilde{f}_{1,2}}^2 \sim M_{\rm SUSY}^2 \pm v M_{\rm SUSY}$. Therefore, in order to establish sfermion masses which respect the lower bounds from direct searches, a hierarchy $M_{\rm SUSY} \gg v$ is needed to a certain degree. In practice it is then sufficient to work to leading order in $v/M_{\rm SUSY}$[5]. This simplifies the expressions for the self-energies and will later allow us to give analytic formulae for the effective vertices.

The leading terms in the expansion of the self-energies in $v/M_{\rm SUSY}$ do not vanish in the limit of infinitely heavy SUSY masses (if all dimensionful SUSY parameters are rescaled

[5] Only in the case of very light SUSY masses, negative μ (which is disfavored by the anomalous magnetic moment of the muon) and large $\tan\beta$, big corrections (compared to the decoupling limit) in the relation between the bottom-quark Yukawa coupling and its mass are possible.

simultaneously). We refer to the approximation in which only such non-decoupling terms are kept as "the decoupling limit". Note, however, that even when working only to leading order in $v/M_{\rm SUSY}$ we do not integrate out the SUSY particles. We rather work in the framework of Refs. [7, 21, 24, 25] in which the SUSY-particles are kept as dynamical degrees of freedom and which thus permits a consistent formulation of effective couplings involving these particles.

To leading order in $v/M_{\rm SUSY}$, the chirality-flipping elements $\Delta^{f\,LR}$ can be neglected in the determination of sfermion mixing matrices. The sfermion mass matrices are then block-diagonal and diagonalized by the mixing matrices W^f:

$$\begin{aligned}
W^{d\dagger}\,\mathcal{M}_d^2\,W^d &= \mathrm{diag}(m_{\tilde{q}_1^L}, m_{\tilde{q}_2^L}, m_{\tilde{q}_3^L}, m_{\tilde{d}_1^R}, m_{\tilde{d}_2^R}, m_{\tilde{d}_3^R})\,, \qquad W^d = \begin{pmatrix} W^{d\,L} & 0 \\ 0 & W^{d\,R} \end{pmatrix}, \\
W^{u\dagger}\,\mathcal{M}_u^2\,W^u &= \mathrm{diag}(m_{\tilde{q}_1^L}, m_{\tilde{q}_2^L}, m_{\tilde{q}_3^L}, m_{\tilde{u}_1^R}, m_{\tilde{u}_2^R}, m_{\tilde{u}_3^R})\,, \qquad W^u = \begin{pmatrix} W^{u\,L} & 0 \\ 0 & W^{u\,R} \end{pmatrix}, \\
W^{\ell\dagger}\,\mathcal{M}_\ell^2\,W^\ell &= \mathrm{diag}(m_{\tilde{\ell}_1^L}, m_{\tilde{\ell}_2^L}, m_{\tilde{\ell}_3^L}, m_{\tilde{\ell}_1^R}, m_{\tilde{\ell}_2^R}, m_{\tilde{\ell}_3^R})\,, \qquad W^\ell = \begin{pmatrix} W^{\ell\,L} & 0 \\ 0 & W^{\ell\,R} \end{pmatrix}. \quad (7)
\end{aligned}$$

The 3×3-matrices $W^{f\,L,R}$ ($f = u, d, \ell$) take into account the flavor mixing in the left- and right-sector of sfermions, respectively. Note that $SU(2)_L$-invariance enforces $\Delta^{u\,LL} = V^{(0)}\,\Delta^{d\,LL}\,V^{(0)\dagger}$. Here $V^{(0)}$ denotes the bare CKM matrix appearing in the diagonalization of the fundamental Yukawa couplings $Y^{u\,(0)}$, $Y^{d\,(0)}$. As a consequence, the masses $m_{\tilde{q}_L^i}$ of left-handed squarks are the same in the up- and down-sector and the corresponding mixing matrices are related to each other via the CKM matrix $V^{(0)}$:

$$W^{d\,L} = W^{q\,L}\,, \qquad\qquad W^{u\,L} = V^{(0)} W^{q\,L}\,. \quad (8)$$

It is further convenient to introduce the abbreviations

$$\begin{aligned}
\Lambda^{f\,LL}_{m\,ij} &= (W^{f\,L})_{im}\,(W^{f\,L\star})_{jm}\,, \qquad (f = u, d, q, \ell), \\
\Lambda^{f\,RR}_{m\,ij} &= (W^{f\,R})_{im}\,(W^{f\,R\star})_{jm}\,, \qquad (f = u, d, \ell), \quad (9)
\end{aligned}$$

where $i, j, m = 1, 2, 3$ and where index m is not summed over.

Left-right-mixing of sfermions, on the other hand, is not described by a mixing matrix but rather treated perturbatively in the form of two-point $\tilde{f}_i^R$-$\tilde{f}_j^L$ vertices governed by the couplings $\Delta^{f\,LR}_{ji}$.

B. Explicit expressions for the self-energies

To leading order in $v/M_{\rm SUSY}$, the self-energy with a gluino and a squark as virtual particles is proportional to one element $\Delta^{q\,LR}_{jk}$ of the squark mixing matrix (note that the self-energy

scales like $\Delta^{q\,LR}_{jk}/M_{SUSY}$ and thus the combination is non-decoupling). We have

$$\begin{aligned}
\Sigma^{d\tilde{g}\,LR}_{fi} &= \frac{2\alpha_s}{3\pi}\, m_{\tilde{g}} \sum_{j,k=1}^{3} \sum_{m,n=1}^{3} \Lambda^{q\,LL}_{m\,fj}\; \Delta^{d\,LR}_{jk}\; \Lambda^{d\,RR}_{n\,ki}\; C_0\Big(m^2_{\tilde{g}}, m^2_{\tilde{q}^L_m}, m^2_{\tilde{d}^R_n}\Big)\,, \\
\Sigma^{u\tilde{g}\,LR}_{fi} &= \frac{2\alpha_s}{3\pi}\, m_{\tilde{g}} \sum_{j,k,j',f'=1}^{3} \sum_{m,n=1}^{3} V^{(0)}_{ff'}\; \Lambda^{q\,LL}_{m\,f'j'}\; V^{(0)\star}_{jj'}\; \Delta^{u\,LR}_{jk}\; \Lambda^{u\,RR}_{n\,ki}\; C_0\Big(m^2_{\tilde{g}}, m^2_{\tilde{q}^L_m}, m^2_{\tilde{u}^R_n}\Big)\,. \quad (10)
\end{aligned}$$

The matrices $\Lambda^{q\,LL,RR}_{m\,ki}$ $(q = u, d)$ take into account all powers of chirality-conserving flavor changes induced through the off-diagonal elements $\Delta^{q\,LL,RR}_{ij}$. For example $\Sigma^{d\,LR}_{11}$ also contains a contribution which, in the mass insertion approximation, would be $\propto \left(\Delta^{d\,LL}_{13}\; \Delta^{d\,LR}_{33}\; \Delta^{d\,RR}_{31}\right)$. Therefore, Eq. (10) is exact in the decoupling-limit. The corresponding self-energy with flipped chiralities is determined through Eq. (2).

For the neutralino-sfermion contributions to the lepton and quark self-energies we get

$$\begin{aligned}
\Sigma^{\ell\tilde{\chi}^0\,LR}_{fi} &= \frac{1}{16\pi^2}\Bigg\{ \sum_{j,k=1}^{3}\sum_{m,n=1}^{3} g_1^2 M_1 \left(\Lambda^{\ell\,LL}_{m\,fj}\;\; \Delta^{\ell\,LR}_{jk}\;\; \Lambda^{\ell\,RR}_{n\,ki}\right) C_0\Big(|M_1|^2, m^2_{\tilde{\ell}^L_m}, m^2_{\tilde{\ell}^R_n}\Big) \\
&+ \sum_{m=1}^{3}\Bigg[\frac{1}{\sqrt{2}g_2} M_W \sin\beta\; Y^{\ell_f(0)} \Lambda^{\ell\,LL}_{m\,fi}\Big(g_2^2 M_2\mu\; C_0\left(|M_2|^2, |\mu|^2, m^2_{\tilde{\ell}^L_m}\right) \\
&- g_1^2 M_1\mu C_0\left(|M_1|^2, |\mu|^2, m^2_{\tilde{\ell}^L_m}\right)\Big) \\
&+ g_1^2 v_u M_1\mu\; Y^{\ell_i(0)} \Lambda^{\ell\,RR}_{m\,ij}\; C_0\left(|M_1|^2, |\mu|^2, m^2_{\tilde{\ell}^R_m}\right)\Bigg]\Bigg\}\,, \\
\Sigma^{d\tilde{\chi}^0\,LR}_{fi} &= \frac{1}{16\pi^2}\Bigg\{ \sum_{j,k=1}^{3}\sum_{m,n=1}^{3} -\frac{1}{9} g_1^2 M_1 \left(\Lambda^{q\,LL}_{m\,fj}\;\; \Delta^{d\,LR}_{jk}\; \Lambda^{d\,RR}_{n\,ki}\right) C_0\Big(|M_1|^2, m^2_{\tilde{q}^L_m}, m^2_{\tilde{d}^R_n}\Big) \\
&+ \sum_{m=1}^{3}\Bigg[\frac{1}{\sqrt{2}g_2} M_W \sin\beta\; Y^{d_i(0)} \Lambda^{q\,LL}_{m\,fi}\Big(g_2^2 M_2\; \mu\; C_0\left(|M_2|^2, |\mu|^2, m^2_{\tilde{q}^L_m}\right) \\
&+ \frac{g_1^2}{3} M_1\mu C_0\left(|M_1|^2, |\mu|^2, m^2_{\tilde{q}^L_m}\right)\Big) \\
&+ \frac{1}{3} g_1^2 v_u M_1\mu\; Y^{d_f(0)} \Lambda^{d\,RR}_{m\,fi}\; C_0\left(|M_1|^2, |\mu|^2, m^2_{\tilde{d}^R_m}\right)\Bigg]\Bigg\}\,, \\
\Sigma^{u\tilde{\chi}^0\,LR}_{fi} &= \frac{1}{16\pi^2} \sum_{m,n=1}^{3} \frac{2}{9} g_1^2 M_1\; V^{(0)}_{ff'}\; \Lambda^{q\,LL}_{m\,f'j'} V^{(0)\star}_{jj'}\;\; \Delta^{u\,LR}_{jk}\;\; \Lambda^{u\,RR}_{n\,ki} C_0\Big(|M_1|^2, m^2_{\tilde{q}^L_m}, m^2_{\tilde{u}^R_n}\Big)\,. \quad (11)
\end{aligned}$$

Finally the chargino-sfermion contributions to the lepton and down-quark self-energy are

given by

$$\begin{aligned}
\Sigma_{fi}^{d\tilde{\chi}^\pm\,LR} &= -\frac{Y^{d_i(0)}}{16\pi^2}\mu\left[\delta_{i3}\,Y^{u_3(0)\star}\sum_{m,n=1}^{3} V_{3f}^{(0)\star}\,\Lambda_{m\,33}^{q\,LL}\,V_{33}^{(0)}\,\Delta_{33}^{u\,LR\star}\,\Lambda_{n\,33}^{u\,RR}\,C_0\Big(|\mu|^2, m^2_{\tilde{q}^L_m}, m^2_{\tilde{u}^R_n}\Big)\right.\\
&\quad \left. - v_u M_2 \sum_{m=1}^{3}\Lambda_{m\,fi}^{q\,LL} C_0\Big(m^2_{\tilde{q}^L_m}, |\mu|^2, |M_2|^2\Big)\right],\\
\Sigma_{fi}^{\ell\tilde{\chi}^\pm\,LR} &= \frac{Y^{\ell_i(0)}}{16\pi^2}\mu v_u M_2 \sum_{m=1}^{3}\Lambda_{m\,fi}^{\ell\,LL} C_0\Big(m^2_{\tilde{\ell}^L_m}, |\mu|^2, |M_2|^2\Big),
\end{aligned} \tag{12}$$

where we have further neglected the small up-type Yukawa couplings of the first two generations and multiple flavor-changes. Chargino contributions to up-quark self-energies cannot be chirally enhanced: a $\tan\beta$ - enhancement is not possible for up-type self-energies since the tree-level up-quark masses are not suppressed by $\cos\beta$ (in contrast to the down-quark ones). An $A_{ij}^{(\prime)d}/(Y_{ij}^u M_{\rm SUSY})$ - enhancement, on the other hand, is neither possible for the third generation, where the large top Yukawa coupling prevents such an effect, nor for the first two generations, where the contribution is suppressed by a small down-type coupling Y^{d_i} $(i = 1, 2)$. Note further that we have neglected terms proportional to $\cot\beta$ in the chargino- and neutralino mass matrices[6].

We denote the sum of all contributions as

$$\begin{aligned}
\Sigma_{fi}^{u\,LR} &= \Sigma_{fi}^{u\tilde{g}\,LR} + \Sigma_{fi}^{u\tilde{\chi}^0\,LR},\\
\Sigma_{fi}^{d\,LR} &= \Sigma_{fi}^{d\tilde{g}\,LR} + \Sigma_{fi}^{d\tilde{\chi}^0\,LR} + \Sigma_{fi}^{d\tilde{\chi}^\pm\,LR},\\
\Sigma_{fi}^{\ell\,LR} &= \Sigma_{fi}^{\ell\tilde{\chi}^0\,LR} + \Sigma_{fi}^{\ell\tilde{\chi}^\pm\,LR}.
\end{aligned} \tag{13}$$

In order to simplify the notation it is useful to define the quantity

$$\sigma_{ji}^{f} = \frac{\Sigma_{ji}^{f\,LR}}{\max\{m_{f_j}, m_{f_i}\}}\,. \tag{14}$$

Here m_{f_i} is the $\overline{\rm MS}$ renormalized quark mass extracted from experiment using the SM prescription. It has to be evaluated at the same scale as the self-energy $\Sigma_{ji}^{f\,LR}$. The ratio σ_{ji}^{f} is a measure of the chiral enhancement of the self-energies with respect to corresponding quark masses.

For the renormalization of the Yukawa couplings and the CKM matrix it is important to distinguish between the parts of $\Sigma_{ji}^{f\,LR}$ which contain a Yukawa coupling and/or CKM

[6] If one would keep the $\cot\beta$ - suppressed terms in the chargino- and neutralino mass matrices, the self-energies would be divergent and one would have to go through the procedure of infinite renormalization. In addition, one would have to consider also the chirally conserving self-energies $\not{p}\Sigma_{ij}^{f\,LL,RR}(0)$ since they generate, after application of the Dirac equation, fermion mass terms of the same order in $m_b/M_{\rm SUSY}$ and in $\tan\beta$ as the $\cot\beta$-suppressed parts of $\Sigma_{ij}^{f\,LR}$.

element and those which do not. Furthermore, for the determination of the effective Higgs-fermion-fermion vertices one has to distinguish between parts of $\Sigma^{f\,LR}_{ji}$ proportional to different Higgs vev's (we call terms in $\Sigma^{d(u)\,LR}_{ji}$ proportional to $v_{d(u)}$ to be "holomorphic", whereas terms in $\Sigma^{d(u)\,LR}_{ji}$ proportional to $v_{u(d)}$ are called "non-holomorphic"). Therefore we will define several corresponding decompositions of $\Sigma^{f\,LR}_{ji}$ (or σ^{f}_{ji}).

In the expressions (10)-(12) each term in the down-quark (lepton) self-energy $\Sigma^{d(\ell)\,LR}_{fi}$ involves at most one power of the corresponding Yukawa-coupling $Y^{d(\ell)}$. The up-quark self-energy $\Sigma^{u\,LR}_{fi}$, on the other hand, is approximatly independent of Y^u as it is always multiplied by $\cot\beta$ and can be neglected if one takes into account only chirally-enhanced contributions. We make the $Y^{d(\ell)}$-dependence of the flavor-conserving self-energy $\Sigma^{d(\ell)\,LR}_{ii}$ explicit by decomposing it as

$$\Sigma^{d(\ell)\,LR}_{ii} \;=\; \Sigma^{d(\ell)\,LR}_{ii\not Y_i} \;+\; \epsilon^{d(\ell)}_i \, v_u \; Y^{d_i(\ell_i)(0)} \,. \tag{15}$$

In a similar way we decompose the flavor-changing self-energies $\Sigma^{q\,LR}_{fi}$ $(q = u, d)$ with respect to CKM elements. Concerning the down-type quarks, only $\Sigma^{d\,LR}_{f3}$ $(f = 1, 2)$ depends on (off-diagonal) CKM elements in the approximation in which we neglect small mass ratios and multiple flavor-changes. For $f \neq i$ we write the enhancement factors σ^d_{fi} as

$$\sigma^d_{fi} \;=\; \begin{cases} \widehat{\sigma}^d_{f3} \;+\; \epsilon^d_{\rm FC} V^{(0)\star}_{3f} V^{(0)}_{33} \,, & \text{i=3} \\ \widehat{\sigma}^d_{fi} \,, & \text{i=1,2} \end{cases} , \tag{16}$$

so that the $\widehat{\sigma}^d_{fi}$ do not depend on (off-diagonal) CKM elements and

$$\varepsilon^d_{FC} = \frac{-1}{16\pi^2}\,\mu\,\frac{Y^{d_3(0)}}{m_{d_3}} \sum_{m,n=1}^{3} Y^{u_3(0)\star}\,\Lambda^{q\,LL}_{m\,33}\,\Delta^{u\,LR\star}_{33}\,\Lambda^{u\,RR}_{n\,33}\,C_0\left(|\mu|^2, m^2_{\tilde{q}^L_m}, m^2_{\tilde{u}^R_n}\right). \tag{17}$$

For the up-quark self-energy $\Sigma^{u\,LR}_{fi}$ the situation is more involved. It depends on the CKM matrix through $\Lambda^{u\,LL}$, which is related to $\Lambda^{q\,LL}$ via the SU(2) relation $\Lambda^{u\,LL} = V^{(0)\dagger}\Lambda^{q\,LL}V^{(0)}$ in the decoupling limit. Therefore, the bare CKM matrix enters the gluino- and neutralino-contributions to $\Sigma^{u\,LR}_{fi}$ in Eqs. (10) and (11). However, there are several reasons why its effect is usually very small. Firstly, a self-energy diagram with an external top quark cannot be significantly chirally enhanced as it has to be compared to the large top quark mass. Furthermore, effects of the CKM matrix in $\Sigma^{u\,LR}_{fi}$ are proportional to the mass splitting of left-handed squarks (and cancel completely if the left-handed squark masses are degenerate[7]). Therefore, in most cases it is an excellent approximation to assume that the up-quark self-energies do not depend on (bare) CKM elements and one can set the CKM elements $V^{(0)}_{ij}$ in Eqs. (10) and (11) to their physical values V_{ij}. We make this approximation explicit by writing

$$\sigma^u_{fi} \;\approx\; \widehat{\sigma}^u_{fi} \tag{18}$$

[7] See Ref. [28] for a discussion of the possibility of non-degenerate squark masses

where $\widehat{\sigma}^u_{fi}$ is understood to be independent of (off-diagonal) bare CKM elements. For completeness in Appendix C we give analytic expressions for the CKM matrix renormalization which take into account the dependence of the up-squark sector on the CKM elements.

For the discussion of the effective Higgs vertices in Sec. IV B we also need a decomposition of $\Sigma^{f\,LR}_{ji}$ into its holomorphic and non-holomorphic parts, as mentioned above. In the decoupling limit all holomorphic self-energies are proportional to A-terms. Thus we denote the holomorphic part as $\Sigma^{f\,LR}_{ji\,A}$, while the non-holomorphic part is denoted as $\Sigma'^{f\,LR}_{ji}$. Then we have

$$\Sigma^{f\,LR}_{ji} = \Sigma^{f\,LR}_{ji\,A} + \Sigma'^{f\,LR}_{ji}, \tag{19}$$

and the corresponding equation for $\sigma^{f\,LR}_{ji}$.

For the decomposition of the self-energies we have assumed that the A-terms and the bilinear soft squark mass terms do not depend on CKM elements or Yukawa couplings. For example in symmetry-based MFV [2] this is not the case and those parameters carry an additional dependence on CKM elements and Yukawa couplings. Then the self-energies are no longer linear in the Yukawa couplings and an analytic resummation, as we will perform in the following chapter, is impossible. In such cases one has to rely on an iterative procedure in order to determine the bare Yukawa couplings and bare CKM elements[8].

III. RENORMALIZATION

In this chapter we consider the general effects of the finite chirally-enhanced self-energies on mass and wave-function renormalization of fermions and on the renormalization of the CKM matrix. We do not consider the renormalization of the PMNS matrix because the renormalization effects are known to be very small [22, 29].

A. Renormalization of fermion masses and Yukawa couplings

Chirally-enhanced self-energies modify the relation between the bare Yukawa couplings $Y^{f_i(0)}$ and the corresponding physical fermion masses m_{f_i}. In our discussion we concentrate on the quark case postponing conclusions for the lepton case to the end of this section. Considering only chirally-enhanced corrections, the physical quark mass is given by

$$m_{q_i} \;=\; v_q Y^{q_i(0)} \;+\; \Sigma^{q\,LR}_{ii}, \qquad (q = u, d). \tag{20}$$

Eq. (20) implicitly determines the bare Yukawa couplings $Y^{q_i(0)}$ for a given set of SUSY parameters. The actual values and physical meaning of the renormalized Y^{q_i} depend, of course, on the renormalization scheme chosen for Y^{q_i}. Thus, to finite order in perturbation theory, the Feynman amplitude for a given process would depend on the chosen scheme.

[8] Iteration is also needed if the results of Ref. [16] are applied to the general MSSM because in [16] the soft SUSY-breaking terms are parametrized in terms of Yukawa couplings.

However, in all-order resummed expressions the scheme dependence drops out and the final results only depend on the (finite) bare Yukawa couplings $Y^{q_i(0)}$, which are scheme independent[9].

The self-energy on the right-hand side of Eq. (20) can in principle contain arbitrarily many powers of Yukawa couplings. Therefore, an analytic solution of Eq. (20) for $Y^{q_i(0)}$ is not possible in the general case. However, since the terms in $\Sigma_{ii}^{q\,LR}$ with higher powers of $Y^{q_i(0)}$ are suppressed by higher powers of $v/M_{\rm SUSY}$, a numerical solution of Eq. (20) can be easily achieved using iterative methods. For a detailed description of such procedure in the MFV case we refer to Ref. [21]. It is obviously still useful to have an approximate analytic formula at hand, and we derive it using the decoupling limit.

In the up-quark sector the enhanced terms in the self-energy $\Sigma_{ii}^{u\,LR}$ are independent of $Y^{u_i(0)}$. Therefore Eq. (20) is easily solved for $Y^{u_i(0)}$ and one finds

$$Y^{u_i(0)} = \left(m_{u_i} - \Sigma_{ii}^{u\,LR}\right)/v_u. \tag{21}$$

In the down-quark sector, if we restrict ourselves to the decoupling limit where we have terms proportional to one power of $Y^{d_i(0)}$ at most, we recover the well-known resummation formula for $\tan\beta$-enhanced corrections, with an extra correction due to the A-terms. The resummation formula is given by

$$Y^{d_i(0)} = \frac{m_{d_i} - \Sigma_{ii\not{Y}_i}^{d\,LR}}{v_d\left(1+\tan\beta\varepsilon_i^d\right)} \tag{22}$$

with ϵ_i^d and $\Sigma_{ii\not{Y}_i}^{d\,LR}$ defined through Eq. (15).

Finally, we note that all statements of this section concerning down-quarks can directly be transferred to the lepton sector. In particular the Yukawa coupling $Y^{\ell_i(0)}$ is obtained from Eq. (22) by replacing fermion index d for ℓ, except for the vev.

B. Fermion wave-function renormalization

The flavor-changing self-energies $\Sigma_{fi}^{f\,LR}$ induce wave-function rotations

$$\psi_i^{f\,L} \longrightarrow U_{ij}^{f\,L}\,\psi_j^{f\,L}\,, \qquad\qquad \psi_i^{f\,R} \longrightarrow U_{ij}^{f\,R}\,\psi_j^{f\,R} \tag{23}$$

in flavor-space which have to be applied to all external fermion fields. We decompose $U_{ij}^{f\,L,R}$ as

$$U_{ij}^{L,R} = \delta_{ij} + \Delta U_{ij}^{q\,L,R\,(1)} + \Delta U_{ij}^{q\,L,R\,(2)} + ... \tag{24}$$

[9] Even though the bare Yukawa couplings $Y^{q_i(0)}$ are independent of the renormalization scheme applied to Y^{q_i}, their values depend on the choice of the SUSY input parameters, i.e. on the renormalization scheme chosen in the squark sector [21].

where the superscripts denote the respective loop order. At the one-loop level $\Delta U_{fi}^{q\,L}$ is given by [23]

$$\Delta U^{f\,L\,(1)} = \begin{pmatrix} 0 & \sigma_{12}^{f} + \frac{m_{f_1}}{m_{f_2}}\,\sigma_{21}^{f\star} & \sigma_{13}^{f} + \frac{m_{f_1}}{m_{f_3}}\,\sigma_{31}^{f\star} \\ -\sigma_{12}^{f\star} - \frac{m_{f_1}}{m_{f_2}}\,\sigma_{21}^{f} & 0 & \sigma_{23}^{f} + \frac{m_{f_2}}{m_{f_3}}\,\sigma_{32}^{f\star} \\ -\sigma_{13}^{f\star} - \frac{m_{f_1}}{m_{f_3}}\,\sigma_{31}^{f} & -\sigma_{23}^{f\star} - \frac{m_{f_2}}{m_{f_3}}\,\sigma_{32}^{f} & 0 \end{pmatrix}, \tag{25}$$

where we have neglected terms which are quadratic or of higher order in small quark mass ratios. However, for transitions between the third and the first generation also two-loop corrections are important [23, 29]. They read

$$\Delta U^{f\,L\,(2)} = \begin{pmatrix} -\frac{1}{2}\left|\sigma_{12}^{f}\right|^2 - \frac{1}{2}\left|\sigma_{13}^{f}\right|^2 & -\frac{m_{f_3}}{m_{f_2}}\,\sigma_{13}^{f}\,\sigma_{32}^{f} & \frac{m_{f_2}}{m_{f_3}}\,\sigma_{12}^{f}\,\sigma_{32}^{f\star} \\ \frac{m_{f_3}}{m_{f_2}}\,\sigma_{13}^{f\star}\,\sigma_{32}^{f\star} & -\frac{1}{2}\left|\sigma_{12}^{f}\right|^2 - \frac{1}{2}\left|\sigma_{23}^{f}\right|^2 & \frac{m_{f_2}}{m_{f_3}}\,\sigma_{21}^{f}\,\sigma_{31}^{f\star} \\ \sigma_{12}^{f\star}\,\sigma_{23}^{f\star} & -\sigma_{12}^{f}\,\sigma_{13}^{f\star} & -\frac{1}{2}\left|\sigma_{13}^{f}\right|^2 - \frac{1}{2}\left|\sigma_{23}^{f}\right|^2 \end{pmatrix}. \tag{26}$$

Here only the leading order in the expansion in small quark mass ratios has been taken into account. Respecting naturalness constraints for the CKM hierarchy, only the $3-1$ element in Eq. (26) can be numerically important. To leading order in the quark mass ratios the full $U^{f\,L}$ then reads

$$U^{f\,L} = \begin{pmatrix} 1 & \sigma_{12}^{f} & \sigma_{13}^{f} \\ -\sigma_{12}^{f\star} & 1 & \sigma_{23}^{f} \\ -\left(\sigma_{13}^{f\star} - \sigma_{12}^{f\star}\,\sigma_{23}^{f\star}\right) & -\sigma_{23}^{f\star} & 1 \end{pmatrix}. \tag{27}$$

The corresponding expressions for $U^{f\,R}$ are obtained from the ones for $U^{f\,L}$ by replacing $\sigma_{ji}^{f} \to \sigma_{ij}^{f\star}$.

C. Renormalization of the CKM matrix

Application of the rotations in Eq. (27) to the $u_i d_j W^+$ - vertex renormalizes the CKM elements V_{ij}. The bare CKM matrix $V^{(0)}$ (stemming from the misalignment between the Yukawa matrices $Y^{u(0)}$ and $Y^{d(0)}$) can be calculated in terms of the physical CKM matrix V as

$$V^{(0)} = U^{u\,L}\,V\,U^{d\,L\dagger}. \tag{28}$$

In the absence of large unnatural cancellations, the rotations $U^{u\,L}$ and $U^{d\,L}$ preserve the hierarchy of V so that $V^{(0)}$ has the same hierarchy as V. However, the conventional Wolfenstein parametrization is not sufficient to describe $U^{u\,L}$, $U^{d\,L}$ and $V^{(0)}$ since these matrices

can have additional complex phases compared to the physical CKM matrix V (in the case of V such phases are absorbed by proper redefinition of the quark fields). Therefore, we extend the classic Wolfenstein parametrization in Appendix A. In terms of our generalized Wolfenstein parametrization, defined in Eq. (A.2), we have

$$V = V\left(\lambda,\, \lambda^2 A,\, \lambda^3 A(\rho - i\eta),\, 0\right) \equiv V\left(v_{12},\, v_{23},\, v_{13},\, 0\right) \tag{29}$$

and

$$U^{q\,L} \;=\; U^{q\,L}\left(\sigma^q_{12},\, \sigma^q_{23},\, \sigma^q_{13},\, 0\right) \qquad\qquad (q = u, d). \tag{30}$$

We parametrize $V^{(0)}$ accordingly as

$$V^{(0)} \;=\; \left(v^{(0)}_{12},\, v^{(0)}_{23},\, v^{(0)}_{13},\, v^{(0)}_{\rm Im}\right). \tag{31}$$

Using the approximation (18), the rotation matrix $U^{u\,L}$ is independent of $V^{(0)}$. We make this explicit by writing

$$U^{u\,L} \;=\; \widehat{U}^{u\,L} \;=\; \widehat{U}^{u\,L}\left(\widehat{\sigma}^u_{12},\, \widehat{\sigma}^u_{23},\, \widehat{\sigma}^u_{13},\, 0\right). \tag{32}$$

The matrix $U^{d\,L}$, on the other hand, consists of a CKM-dependent and a CKM-independent part since the σ^d_{ji} entering Eq. (27) decompose according to Eq. (16). We transfer this decomposition to $U^{d\,L}$ writing (in what follows we neglect terms $\mathcal{O}(\lambda^4)$ and higher)

$$U^{d\,L} \;=\; U^{d\,L}_{\rm CKM}\, \widehat{U}^{d\,L}. \tag{33}$$

The CKM-independent part $\widehat{U}^{d\,L}$ is defined by replacing $\sigma^d_{ji} \to \widehat{\sigma}^d_{ji}$ in Eq. (27), what amounts to the generalized Wolfenstein parametrization

$$\widehat{U}^{d\,L} \;=\; \widehat{U}^{d\,L}\left(\widehat{\sigma}^d_{12},\, \widehat{\sigma}^d_{23},\, \widehat{\sigma}^d_{13},\, 0\right). \tag{34}$$

The CKM-dependent part $U^{d\,L}_{\rm CKM}$ is then given by

$$\begin{aligned} U^{d\,L}_{\rm CKM} \;&=\; U^{d\,L}\,\widehat{U}^{d\,L\dagger} \;=\; \begin{pmatrix} 1 & 0 & V^{(0)\star}_{31} V^{(0)}_{33}\, \varepsilon^d_{FC} \\ 0 & 1 & V^{(0)\star}_{32} V^{(0)}_{33}\, \varepsilon^d_{FC} \\ -V^{(0)}_{31} V^{(0)\star}_{33}\, \varepsilon^{d\star}_{FC} & -V^{(0)}_{32} V^{(0)\star}_{33}\, \varepsilon^{d\star}_{FC} & 1 \end{pmatrix} \\ &=\; U^{d\,L}_{\rm CKM}\left(0,\, V^{(0)\star}_{32} V^{(0)}_{33}\, \varepsilon^d_{FC},\, V^{(0)\star}_{31} V^{(0)}_{33}\, \varepsilon^d_{FC},\, 0\right). \end{aligned} \tag{35}$$

Inserting the decomposition (33) into Eq. (28) we obtain

$$V^{(0)} \;=\; \left(\widehat{U}^{u\,L}\, V\, \widehat{U}^{d\,L\dagger}\right) U^{d\,L\dagger}_{\rm CKM}. \tag{36}$$

In order to determine $V^{(0)}$, we have to solve Eq. (36). The right-hand side implicitly depends

on $V^{(0)}$ through $U^{dL}_{\rm CKM}$. As a first step we solve Eq. (36) in the special case $U^{dL}_{\rm CKM} \equiv 1$, i.e. in the absence of the contributions governed by ε^d_{FC}. This means we calculate

$$\widetilde{V} \,=\, \widehat{U}^{u\,L}\, V\, \widehat{U}^{d\,L\dagger}\,. \tag{37}$$

Exploiting the multiplication rule (A.7) for generalized Wolfenstein matrices we get

$$\widetilde{V} \,=\, (\widetilde{v}_{12},\, \widetilde{v}_{23},\, \widetilde{v}_{13},\, \widetilde{v}_{\rm Im}) \tag{38}$$

with

$$\begin{aligned}
\widetilde{v}_{12} \,&=\, v_{12} \,+\, \widehat{\sigma}^u_{12} \,-\, \widehat{\sigma}^d_{12}\,, \qquad\qquad \widetilde{v}_{23} \,=\, v_{23} \,+\, \widehat{\sigma}^u_{23} \,-\, \widehat{\sigma}^d_{23}\,,\\
\widetilde{v}_{13} \,&=\, v_{13} \,+\, \widehat{\sigma}^u_{13} \,-\, \widehat{\sigma}^d_{13} \,+\, \widehat{\sigma}^u_{12} v_{23} \,+\, \left(\widehat{\sigma}^d_{12} \,-\, \widehat{\sigma}^u_{12}\right) \widehat{\sigma}^d_{23} \,-\, v_{12} \widehat{\sigma}^d_{23}\,,\\
\widetilde{v}_{\rm Im} \,&=\, v_{12}\, {\rm Im}\left[\widehat{\sigma}^u_{12} \,+\, \widehat{\sigma}^d_{12}\right] \,-\, {\rm Im}\left[\widehat{\sigma}^u_{12} \widehat{\sigma}^{d\star}_{12}\right]\,.
\end{aligned} \tag{39}$$

Switching on $U^{dL}_{\rm CKM} \neq 1$ in a second step and solving Eq. (36) for $V^{(0)}$, we finally find

$$V^{(0)} = V^{(0)} \left(\widetilde{v}_{12},\, \frac{\widetilde{v}_{23}}{1-\varepsilon^d_{FC}},\, \frac{\widetilde{v}_{23}}{1-\varepsilon^d_{FC}},\, \widetilde{v}_{\rm Im}\right). \tag{40}$$

Explicitly written down, this matrix reads

$$V^{(0)} = \begin{pmatrix}
1 - \dfrac{|\widetilde{v}_{12}|^2}{2} + i\,\widetilde{v}_{\rm Im} & \widetilde{v}_{12} & \dfrac{\widetilde{v}_{13}}{1-\varepsilon^d_{FC}} \\
-\widetilde{v}^\star_{12} & 1 - \dfrac{|\widetilde{v}_{12}|^2}{2} - i\,\widetilde{v}_{\rm Im} & \dfrac{\widetilde{v}_{23}}{1-\varepsilon^d_{FC}} \\
\dfrac{\widetilde{v}^\star_{12}\widetilde{v}^\star_{23} - \widetilde{v}^\star_{13}}{1-\varepsilon^{d\star}_{FC}} & -\dfrac{\widetilde{v}^\star_{23}}{1-\varepsilon^{d\star}_{FC}} & 1
\end{pmatrix}. \tag{41}$$

We see that the elements $\widetilde{v}_{13}$ and $\widetilde{v}_{23}$ in Eq. (41) are scaled by a factor $1/(1-\varepsilon^d_{FC})$. This generalizes the observation of ref. [21], where it has been found that the Wolfenstein parameter A is scaled by this factor in the MFV version of the MSSM, to the case of general flavor violation.

D. Proper renormalization sequence

The determination of the bare Yukawa couplings and bare CKM matrix is complicated by the fact that the corresponding equations, defined in the previous sections, are entangled. We give here a detailed recipe on how to determine these quantities step by step.

1. One should start from the calculation of the bare Yukawa couplings.

   a) Calculate the flavor-conserving self-energies $\Sigma^{u\,LR}_{ii}$ in the up-sector from Eq. (10) and Eq. (11). Note that $\Sigma^{u\,LR}_{ii}$ is independent of any Yukawa coupling since we neglect terms proportional to $\cot\beta$. Determine the bare up-quark Yukawa couplings $Y^{u_i(0)}$ via Eq. (21).

b) Having the bare top-quark Yukawa coupling at hand, extract the chargino contribution to ϵ_3^d (defined in Eq. (15)) from Eq. (12). Calculate also all other contributions to ϵ_3^d as well as to the $Y^{d_3(0)}$-independent part $\Sigma^{d\,LR}_{33\not{Y}_3^d}$ of the flavor-conserving self-energy $\Sigma^{d\,LR}_{33}$ from Eqs. (10) - (12). For this step one can neglect small contributions proportional to the strange- or down-Yukawa couplings, which are still undetermined. Calculate the bare bottom Yukawa coupling $Y^{d_3(0)}$ from Eq. (22).

c) Calculate ϵ_2^d and $\Sigma^{d\,LR}_{22\not{Y}_2^d}$ for the strange quark analogously to step 1b) for the bottom quark. In the calculation, $Y^{d_3(0)}$ should be set to the value determined in step 1b), while $Y^{d_1(0)}$ can again be neglected. Compute $Y^{d_2(0)}$ according to Eq. (22).

d) Proceed in the same way for $Y^{d_1(0)}$ using the already determined values for $Y^{d_2(0)}, Y^{d_3(0)}$.

2. In the next step, the bare CKM matrix and the field rotation matrices can be determined.

a) Use the value of the bare Yukawa couplings to determine ϵ_{FC}^d and the CKM-independent self-energy parameters $\widehat{\sigma}_{ij}^u$ and $\widehat{\sigma}_{ij}^d$ from Eqs. (10) - (12) according to the decompositions (16),(18). This allows to compute the bare CKM matrix $V_{ij}^{(0)}$ with help of Eqs. (39) and (41).

b) Next, insert the bare Yukawa couplings and $V_{ij}^{(0)}$ into Eq. (14) in order to compute the full σ_{ij}^d. Also σ_{ij}^u should be recalculated using the bare $V_{ij}^{(0)}$ (instead of the V_{ij} which have been used in the calculation of the $\widehat{\sigma}_{ij}^u$). With the σ_{ij}^u and σ_{ij}^d at hand, one calculates the rotation matrices $U_{ij}^{q\,L,R}$ $(q = u, d)$ in Eq. (27).

The procedure used for the down quarks applies to the charged leptons as well.

IV. EFFECTIVE FERMION VERTICES

Having determined in the previous section the bare Yukawa couplings and the bare CKM matrix we are now in a position to calculate the effective gaugino(higgsino)-fermion-sfermion and the effective Higgs-fermion-fermion vertices in the general MSSM.

A. Effective gaugino-fermion-sfermion and higgsino-fermion-sfermion vertices

In order to calculate the effective gaugino(higgsino)-fermion-sfermion vertices, one has to take the Feynman-rules given in Appendix D and substitute in the couplings $\Gamma^{\tilde{\lambda}\,L,R}_{f_j\tilde{f}_s}$ the tree-level Yukawa couplings and the CKM matrix by the corresponding bare quantities (since the Feynman-rules in Appendix D go beyond the decoupling limit approximation, one should also recalculate the sfermion masses and sfermion mixing matrices with the use of

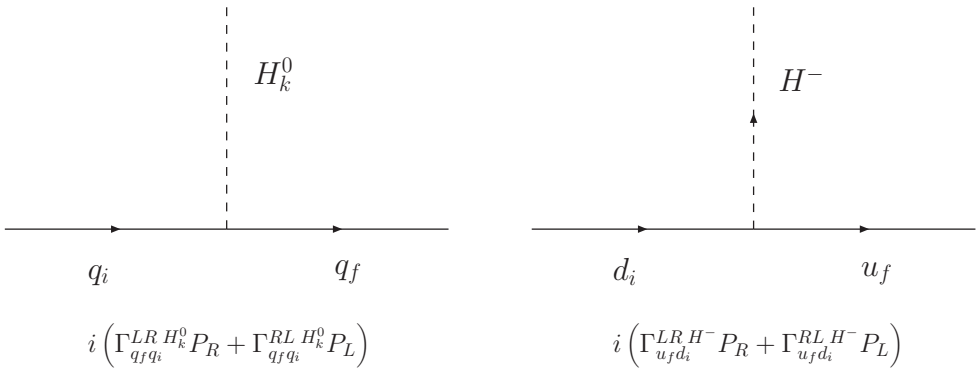

FIG. 2: Higgs-quark vertices with the corresponding Feynman-rules.

bare quantities). In addition, one has to apply the wave-function rotations to the fermion fields replacing $\Gamma^{\tilde{\lambda}\,L,R}_{f_j \tilde{f}_s}$ by

$$\Gamma^{\tilde{\lambda}\,L\,\text{eff}}_{f_i \tilde{f}_s} = U^{f\,L}_{ji}\,\Gamma^{\tilde{\lambda}\,L}_{f_j \tilde{f}_s}, \qquad \Gamma^{\tilde{\lambda}\,R\,\text{eff}}_{f_i \tilde{f}_s} = U^{f\,R}_{ji}\,\Gamma^{\tilde{\lambda}\,R}_{f_j \tilde{f}_s}\,. \tag{42}$$

If the momentum p flowing through the fermion line satisfies $p^2 \ll M^2_{\text{SUSY}}$, the rotations $U^{f\,L,R}_{ji}$ take into account the effects of flavor-changing chirally-enhanced self-energy corrections (to leading order in p^2/M^2_{SUSY}). For $p^2 \sim M^2_{\text{SUSY}}$, on the other hand, no chiral enhancement occurs and the rotations $U^{f\,L,R}_{ji}$ drop out from internal fermion lines. Therefore our effective vertices can be applied irrespective of the momentum flowing through the fermion line.

The appearance of the rotations $U^{f\,L,R}_{ji}$ in gaugino(higgsino)-fermion-sfermion vertices is a consequence of the fact that our super-CKM basis is defined at the level of the bare Yukawa couplings $Y^{q(0)}$. Therefore, it is natural to ask whether these effects can be absorbed into the definition of the squark mass terms if an on-shell definition for the super-CKM basis is used. Note, however, that at least for the higgsino-parts of the chargino- and neutralino-vertices, this is impossible: if an on-shell definition for the super-CKM basis is used, the bare Yukawa couplings $Y^{q(0)}_{ij}$ develop off-diagonal entries which are related to the rotation matrices $U^{f\,L,R}_{ji}$. In this way the physical effects of these rotations would reappear in the higgsino-fermion-sfermion coupling. Note further that an absorption of the effects in gaugino-fermion-sfermion vertices, is only possible as long as the bilinear SUSY breaking terms are independent free parameters. As soon as a structure resulting from a SUSY breaking mechanism (like gravity-mediation or gauge-mediation) is assumed for them, an arbitrary redefinition is not possible anymore and the effects of $U^{f\,L,R}_{ji}$ become physical here as well.

B. Effective Higgs-fermion-fermion vertices

Also Higgs-fermion-fermion couplings receive chirally-enhanced corrections from the Yukawa- and CKM-renormalization and from the fermion wave-function rotations. In addition, we face a new class of chirally-enhanced effects: the Higgs coupling itself involves a

Yukawa coupling Y^f with $Y^f \ll 1$ for $f \neq t$. Therefore a genuine vertex correction which avoids the Y^f - suppression by coupling to the Higgs via the $A^{(\prime)f}$ - term can be chirally enhanced with respect to the tree-level vertex. The loop suppression can be alleviated by a factor $A^{(\prime)f}_{ij}/(Y^f_{ij} M_{\rm SUSY})$ in this case. Note that this type of chiral enhancement cannot replicate itself at higher orders in perturbation theory, so that no resummation is needed.

Since all corrections to gaugino(higgsino)-fermion-sfermion vertices were due to fermion self-energies, they did not depend on the momenta of the SUSY particles but only on the momentum p of the fermion. As shown in Refs. [7, 21], chirally-enhanced effects only occur for $p^2 \ll M^2_{\rm SUSY}$. Therefore, such effects are local and can be cast into effective Feynman rules without any further assumptions. In the case of the genuine vertex corrections to the Higgs-fermion-fermion couplings, the situation is different. These corrections are chirally enhanced, independently of the scale of the external momenta. In order to derive effective Feynman rules for these vertices, however, we have to assume that the external momenta are much smaller than the masses of the virtual SUSY particles running in the loop. This assumption limits the applicability of the resulting Feynman rules: if $m_{H^0}, m_{A^0}, m_{H^\pm} \ll M_{\rm SUSY}$ ($H^0, A^0, H^\pm$ denote the neutral CP-even, CP-odd and the charged Higgs boson, respectively), they can be used for all processes including diagrams where the Higgs bosons are involved in a loop. If this hierarchy is not satisfied, they can only be used for processes in which the momentum-flow through the Higgs-fermion-fermion vertex is small compared to $M_{\rm SUSY}$. Important examples for processes of the latter kind are the Higgs penguins contributing to $B_{d,s} \to \mu^+\mu^-$, $B^+ \to \tau^+\nu$ or the double Higgs penguin contributing to $\Delta F = 2$ processes.

Effective Higgs-fermion-fermion vertices have been calculated in Ref. [25], but only the gluino-squark contributions have been taken into account. We extend the results of Ref. [25] by including also chargino-fermion and neutralino-fermion corrections. In Ref. [25] two different derivations of the effective Higgs vertices have been presented: the first one, using a diagrammatic method, delivers a result valid to all orders in $v/M_{\rm SUSY}$, while the second one, using an effective theory approach, reproduces only the leading order in $v/M_{\rm SUSY}$. It turned out that the leading order in $v/M_{\rm SUSY}$ is an excellent approximation to the full approach and there is no reason why this statement should not be true for the chargino and neutralino contributions. Furthermore, since we restricted ourselves to leading order in $v/M_{\rm SUSY}$ in the resummation of the Yukawa couplings (which enter the Higgs coupling), for consistency we should rely on this approximation in calculating the genuine vertex corrections as well. Therefore, we will use the effective field theory approach in our study of the Higgs-fermion-fermion couplings which simplifies the calculations. This means that in contrast to the previous sections we really integrate out the SUSY particles and remove them as dynamical degrees of freedom, limiting somewhat the applicability of the effective Higgs vertices as discussed in the previous paragraph.

The resulting effective Yukawa-Lagrangian is that of a general 2HDM and we parametrize it (in the super-CKM basis) as

$$\begin{aligned}
\mathcal{L}_Y^{eff} \;=\; & \bar{Q}^a_{f\,L}\left[(Y^{d_i(0)}\delta_{fi} + E^d_{fi})\epsilon_{ab} H_d^{b\star} \;-\; E'^d_{fi} H_u^a\right] d_{i\,R} \\
& -\; \bar{Q}^a_{f\,L}\left[(Y^{u_i(0)}\delta_{fi} + E^u_{fi})\epsilon_{ab} H_u^{b\star} \;+\; E'^u_{fi} H_d^a\right] u_{i\,R} \;+\; h.c.
\end{aligned} \tag{43}$$

Here a, b denote $SU(2)_L$ - indices and ϵ_{ab} is the two-dimensional antisymmetric tensor with $\epsilon_{12} = 1$. Apart from the Yukawa-couplings Y^{u_i} and Y^{d_i}, we have in the effective theory loop-induced holomorphic couplings E^q_{fi} and non-holomorphic couplings E'^q_{fi} ($q = u, d$). In the general MSSM these couplings can be expressed in terms of the corresponding self-energies, which also decompose into a holomorphic and a non-holomorphic part according to Eq. (19). We have

$$E^d_{ij} \;=\; \frac{\Sigma^{d\,LR}_{ij\,A}}{v_d}\,, \qquad E'^d_{ij} \;=\; \frac{\Sigma'^{\,d\,LR}_{ij}}{v_u} \qquad E^u_{ij} \;=\; \frac{\Sigma^{u\,LR}_{ij\,A}}{v_u}\,, \qquad E'^u_{ij} \;=\; \frac{\Sigma'^{\,u\,LR}_{ij}}{v_d}\,. \quad (44)$$

These effective couplings are in principle loop-suppressed compared to the tree-level $Y^{d_i(0)}$, $Y^{u_i(0)}$ but a chiral enhancement of $A^{(\prime)q}_{ij}/(Y^q_{ij} M_{\rm SUSY})$ can compensate for this suppresion.

In our effective theory approach, the wave-function rotations[10] modify the effective Lagrangian as follows [25]:

$$\begin{aligned}
\mathcal{L}^{eff}_Y \;=\; &-\; \bar{d}_{f\,L}\left[\left(\frac{m_{d_i}}{v_d}\delta_{fi} - \widetilde{E}'^d_{fi}\tan\beta\right) H^{0\star}_d \;+\; \widetilde{E}'^d_{fi}\, H^0_u\right] d_{i\,R} \\
&-\; \bar{u}_{f\,L}\left[\left(\frac{m_{u_i}}{v_u}\delta_{fi} - \widetilde{E}'^u_{fi}\cot\beta\right) H^{0\star}_u \;+\; \widetilde{E}'^u_{fi}\, H^0_d\right] u_{i\,R} \\
&+\; \bar{u}_{f\,L} V_{fj}\left[\frac{m_{d_i}}{v_d}\delta_{ji} - (\cot\beta + \tan\beta)\,\widetilde{E}'^d_{ji}\right]\sin\beta H^+ d_{i\,R} \\
&+\; \bar{d}_{f\,L} V^\star_{jf}\left[\frac{m_{u_i}}{v_u}\delta_{ji} - (\tan\beta + \cot\beta)\,\widetilde{E}'^u_{ji}\right]\cos\beta H^- u_{i\,R} \;+\; h.c.
\end{aligned} \quad (45)$$

with

$$\widetilde{E}'^q_{fi} \;=\; U^{q\,L*}_{jf} E'^q_{jk} U^{q\,R}_{ki} \;\approx\; E'^q_{fi} \;-\; \Delta E'^q_{fi}\,,$$

$$\Delta E'^q \;=\; \begin{pmatrix} 0 & \sigma^q_{12} E'^q_{22} & (\sigma^q_{13} - \sigma^q_{12}\sigma^q_{23})\, E'^q_{33} + \sigma^q_{12} E'^q_{23} \\ E'^q_{22}\sigma^q_{21} & 0 & \sigma^q_{23} E'^q_{33} \\ E'^q_{33}\,(\sigma^q_{31} - \sigma^q_{32}\sigma^q_{21}) + E'^q_{32}\sigma^q_{21} & E'^q_{33}\sigma^q_{32} & 0 \end{pmatrix}. \quad (46)$$

The fields H^0_u and H^0_d decompose into the physical components H^0, h^0 and A^0 as

$$\begin{aligned}
H^0_u \;&=\; \frac{1}{\sqrt{2}}\left(H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta\right)\,, \\
H^0_d \;&=\; \frac{1}{\sqrt{2}}\left(H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta\right)\,.
\end{aligned} \quad (47)$$

[10] Note that even though these rotations are identical to the ones in Eq. (27) their origin is different in the effective field theory approach. The matrices $U^{q\,L,R}$ are now obtained by a perturbative diagonalization of the (physical) quark mass matrices (see Ref. [25] for details).

Without the non-holomorphic corrections E'^q_{ij} the rotation matrices $U^{q\,L,R}$ would simultaneously diagonalize the effective mass terms and the neutral Higgs couplings in Eq. (45). However, in the presence of non-holomorphic corrections this is no longer the case and apart from a flavor-changing non-holomorphic correction also a term proportional to a flavor-conserving non-holomorphic correction times a flavor-changing self-energy is generated.

It is instructive to discuss this effect also in the full theory. The two diagrams in Fig. 3 (both involving a holomorphic A-terms) have opposite sign and cancel in the limit $\mu, A'^q \to 0$. However, in the presence of non-holomorphic terms the cancellation is incomplete and a part proportional to $1 - \dfrac{1}{1+\epsilon_b \tan\beta}$ (for down-quarks) survives (second term in Eq. (46)). Even though this term is formally of higher loop order, it is numerically relevant due to its chiral enhancement.

The non-holomorphic parts of the fermion self-energy, as defined in Eq. (19), can be extracted from Eqs. (10), (11) and (12). Note that the whole chargino contribution is always non-holomorphic except for $\cot\beta$ - suppressed terms. The same is true for the neutralino contribution except for the pure bino part which decomposes in the same way as the (dominant) gluino contribution (the latter given already in [25]).

Using Eq. (46) and Eq. (47), the effective Lagrangian in Eq. (45) leads to the following effective Higgs-fermion-fermion Feynman rules[11] (note that the CKM matrix V in the charged Higgs coupling is the physical one):

$$\begin{aligned}
\Gamma^{H^0_k\,LR\,\text{eff}}_{u_f u_i} &= x^k_u \left(\frac{m_{u_i}}{v_u}\delta_{fi} - \widetilde{E}'^u_{fi}\cot\beta \right) + x^{k\star}_d \widetilde{E}'^u_{fi}\,, \\
\Gamma^{H^0_k\,LR\,\text{eff}}_{d_f d_i} &= x^k_d \left(\frac{m_{d_i}}{v_d}\delta_{fi} - \widetilde{E}'^d_{fi}\tan\beta \right) + x^{k\star}_u \widetilde{E}'^d_{fi}\,, \\
\Gamma^{H^\pm\,LR\,\text{eff}}_{u_f d_i} &= \sum_{j=1}^{3} \sin\beta\, V_{fj} \left(\frac{m_{d_i}}{v_d}\delta_{ji} - \widetilde{E}'^d_{ji}\tan\beta \right) \\
\Gamma^{H^\pm\,LR\,\text{eff}}_{d_f u_i} &= \sum_{j=1}^{3} \cos\beta\, V^\star_{jf} \left(\frac{m_{u_i}}{v_u}\delta_{ji} - \widetilde{E}'^u_{ji}\tan\beta \right)\,,
\end{aligned} \tag{48}$$

where for $H^0_k = (H^0, h^0, A^0)$ the coefficients x^k_q are given by[12]

[11] Note that some of the Higgs-quark-quark couplings are suppressed by a factor $\cos\beta$ or $\sin\alpha$ stemming from the Higgs mixing matrices. If one decides to keep these suppressed couplings, one should be aware of the fact that they receive proper vertex corrections in which the suppression factor does not occur and which are thus $\tan\beta$-enhanced with respect to the tree-level couplings. Such enhanced corrections to the coupling of $H^\pm$ to right-handed up-quarks are important for $b \to s\gamma$ [30, 31]. (see Appendix B)

[12] In principle also the renormalization of the Higgs potential should be addressed. Our derivation of chirally enhanced flavor effects does not depend on the specific relations between Higgs self-couplings and their masses. Since no chirally-enhanced effects occur in the Higgs sector, it is consistent to use the tree-level values for the Higgs parameters. However, one can as well use the NLO values for the Higgs masses and mixing angles which might be even better from the numerical point of view.

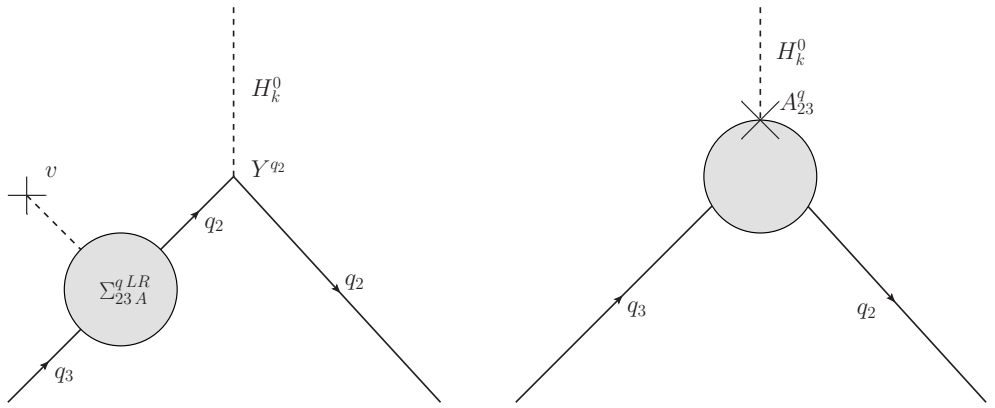

FIG. 3: Self-energy and genuine vertex correction involving A_{23}^q contributing to the effective Higgs coupling.

$$x_d^k = \left(-\frac{1}{\sqrt{2}}\cos\alpha,\ \frac{1}{\sqrt{2}}\sin\alpha,\ \frac{i}{\sqrt{2}}\sin\beta\right), \qquad x_u^k = \left(-\frac{1}{\sqrt{2}}\sin\alpha,\ -\frac{1}{\sqrt{2}}\cos\alpha,\ \frac{i}{\sqrt{2}}\cos\beta\right). \tag{49}$$

It is important to keep in mind that the σ_{ij}^f in Eq. (46) must be calculated using the bare quantities ($Y^{f(0)}$ and $V^{(0)}$).

For the lepton case, the non-vanishing effective Higgs vertices read

$$\begin{aligned}
\Gamma_{\ell_f\ell_i}^{H_k^0\,LR\,\text{eff}} &= x_d^k\left(\frac{m_{\ell_i}}{v_d}\delta_{fi} - \widetilde{E}'^{\ell}_{fi}\tan\beta\right) + x_u^{k\star}\widetilde{E}'^{\ell}_{fi}\,, \\
\Gamma_{\nu_f\ell_i}^{H^\pm\,LR\,\text{eff}} &= \sum_{j=1}^{3}\sin\beta\,V_{fj}^{\text{PMNS}}\left(\frac{m_{\ell_i}}{v_d}\delta_{ji} - \widetilde{E}'^{\ell}_{ji}\tan\beta\right).
\end{aligned} \tag{50}$$

V. CONCLUSIONS

In the general MSSM, chirally-enhanced corrections are induced by gluino-squark, chargino-sfermion and neutralino-sfermion loops and can numerically compete with, or even dominate over, tree-level contributions, due to their enhancement by either $\tan\beta$ or $A_{ij}^f/(Y_{ij}^f M_{\text{SUSY}})$. In this article we have identified all potential sources of chirally-enhanced corrections and discussed their effects on the finite renormalization of Yukawa couplings, fermion wave-functions and the CKM matrix. To leading order in v/M_{SUSY}, which numerically is a very good approximation for realistic choices of MSSM parameters, we obtained analytic resummation formulae for these quantities.

For the CKM resummation, it turned out to be useful to define a generalized Wolfenstein parametrization, obtained by extending the classical one to the case of complex λ and A parameters. This parametrization is presented in Appendix A.

For the resummation of the chirally-enhanced corrections in supersymmetric fermion vertices, we have used the diagrammatic approach developed in Refs. [7, 21, 23, 24]. This method allowed us to cast chirally-enhanced corrections to gaugino(higgsino)-fermion-sfermion couplings into effective vertices, as described in Sec. III D and IV A.

Moreover, we have given formulae for the effective Higgs-fermion-fermion vertices, where we extended the results of [25] by adding the chargino and neutralino contributions. Our effective Higgs-vertices can be used in the limit $m_{H^0}, m_{A^0}, m_{H^\pm} \ll M_{\rm SUSY}$ as Feynman rules in an effective theory with the SUSY particles being integrated out. However, they remain still valid in the case $m_{H^0}, m_{A^0}, m_{H^\pm} \sim M_{\rm SUSY}$ as long as the momenta flowing through the Higgs vertices are much smaller than $M_{\rm SUSY}$. Thus our effective Higgs-fermion-fermion Feynman rules can e.g. be applied to calculate Higgs penguins contributing to $B_{d,s} \to \mu^+\mu^-$, $B^+ \to \tau^+\nu$ or the double Higgs penguin contributing to $\Delta F = 2$ processes.

If our effective matter fermion-sfermion-SUSY fermion and Higgs-fermion-fermion Feynman rules are used for the calculation of an Feynman amplitude at leading order in perturbation theory, all kinds of chirally-enhanced effects are automatically included and resummed to all orders in the result.

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Appendix.

In this Appendix we collect definitions and conventions needed in the article. Further, we define a generalized Wolfenstein parametrization for the bare CKM matrix and give the non-holomorphic parts of the up-quark self-energies with neutralions and charginos as virtual particles.

A. Generalized Wolfenstein Parametrization

While a general unitary 3×3 matrix is described by 3 mixing angles and 6 complex phases, only one of those phases is physical in the case of the CKM matrix V. The other 5 phases are absorbed by proper redefinition of the quark fields exploiting the $U(3)^3$-flavor symmetry of the gauge interactions. After application of this procedure to the physical CKM matrix V, the quark field phases are fixed. As a consequence, possible additional phases in the bare CKM matrix $V^{(0)}$, which originate from the diagonalization of the bare Yukawa couplings $Y^{u(0)}$, $Y^{d(0)}$, cannot be absorbed anymore and thus they are physical[13].

The hierarchical structure of the measured CKM matrix V can be made explicit by using the Wolfenstein parametrization

$$V \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & \lambda^3 A\,(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & \lambda^2 A \\ \lambda^3 A\,(1 - \rho - i\eta) & -\lambda^2 A & 1 \end{pmatrix} \tag{A.1}$$

with the small expansion parameter $\lambda = 0.225$. The three mixing angles and the phase of V are expressed via the four real parameters λ, A, ρ, η. Considering fine-tuning arguments, it is reasonable to assume that the bare CKM matrix $V^{(0)}$ in the general MSSM has a similar hierarchical structure as the physical CKM matrix V. Therefore it is desirable to have a parametrization analogous to Eq. (A.1) but allowing for possible additional phases of $V^{(0)}$.

We will consider the following generalization of the Wolfenstein parametrization:

$$U\left(u_{12}, u_{23}, u_{13}, u_{\mathrm{Im}}\right) = \begin{pmatrix} 1 - \frac{|u_{12}|^2}{2} + i u_{\mathrm{Im}} & u_{12} & u_{13} \\ -u_{12}^\star & 1 - \frac{|u_{12}|^2}{2} - i u_{\mathrm{Im}} & u_{23} \\ -\left(u_{13}^\star - u_{12}^\star u_{23}^\star\right) & -u_{23}^\star & 1 \end{pmatrix}. \tag{A.2}$$

The parameters $u_{12}\,, u_{23}\,, u_{13} \in \mathbb{C}$ and the parameter $u_{\mathrm{Im}} \in \mathbb{R}$ should follow the hierarchical structure of the usual Wolfenstein parametrization (A.1):

$$u_{12} = \mathcal{O}(\lambda), \qquad u_{23} = \mathcal{O}(\lambda^2), \qquad u_{13} = \mathcal{O}(\lambda^3), \qquad u_{\mathrm{Im}} = \mathcal{O}(\lambda^2). \tag{A.3}$$

[13] In principle the additional phases could be absorbed into the wave functions of the bare quark fields $\psi^{(0)}$. However, in this case they would modify the relation between the bare fields $\psi^{(0)}$ and the physical fields ψ and they would enter Feynman amplitudes in the form of complex CP-violating wave-function factors. Since in this case CP violation in the quark sector would not be restricted to the CKM matrix anymore, we refrain from introducing this kind of wave-function rephasing.

Our parametrization is closed under hermitian conjugation and under matrix multiplication. We have (neglecting terms of $\mathcal{O}(\lambda^4)$ and higher)

- hermitian conjugation:

$$U^\dagger(u_{12}, u_{23}, u_{13}, u_{\rm Im}) \;=\; U(\widetilde{u}_{12}, \widetilde{u}_{23}, \widetilde{u}_{13}, \widetilde{u}_{\rm Im}) \tag{A.4}$$

  where

$$\begin{aligned} \widetilde{u}_{12} &= -u_{12}, & \widetilde{u}_{23} &= -u_{23}, \\ \widetilde{u}_{13} &= -\left(u_{13} - u_{12}u_{23}\right), & \widetilde{u}_{\rm Im} &= -u_{\rm Im}\,. \end{aligned} \tag{A.5}$$

- matrix multiplication:

$$U(u''_{12}, u''_{23}, u''_{13}, u''_{\rm Im}) \;=\; U(u_{12}, u_{23}, u_{13}, u_{\rm Im})\;\; U(u'_{12}, u'_{23}, u'_{13}, u'_{\rm Im}) \tag{A.6}$$

  where

$$\begin{aligned} u''_{12} &= u_{12} + u'_{12}, & u''_{23} &= u_{23} + u'_{23}, \\ u''_{13} &= u_{13} + u'_{13} + u_{12}u'_{23}, & u''_{\rm Im} &= u_{\rm Im} + u'_{\rm Im} + {\rm Im}\left[u_{12}u'^{\star}_{12}\right]\,. \end{aligned} \tag{A.7}$$

Note in particular that the parameter $u_{\rm Im}$ had to be introduced in order to make this parametrization closed under multiplication.

We will now demonstrate that our parametrization, which allows for 3 mixing angles and 4 complex phases, can be used to describe the bare CKM matrix $V^{(0)}$ in the MSSM. First we recognize that defining $V = V\left(v_{12}, v_{23}, v_{13}, v_{\rm Im}\right)$ with

$$v_{12} = \lambda, \qquad v_{23} = A\lambda^2, \qquad v_{13} = \lambda^3 A\left(\rho - i\eta\right), \qquad v_{\rm Im} = 0 \tag{A.8}$$

we recover the usual Wolfenstein parametrization (A.1). Furthermore, also the matrix $U^{f\,L}$ given in Eq. (27) can be described in the form $U^{f\,L} = U^{f\,L}(u_{12}^{f\,L}, u_{23}^{f\,L}, u_{13}^{f\,L}, u_{\rm Im}^{f\,L})$:

$$u_{12}^{f\,L} = \sigma_{12}^f, \qquad u_{23}^{f\,L} = \sigma_{23}^f, \qquad u_{13}^{f\,L} = \sigma_{13}^f, \qquad u_{\rm Im}^{f\,L} = 0. \tag{A.9}$$

Because the parametrization is closed under hermitian conjugation and matrix multiplication, the relation between the physical CKM matrix and the bare one in Eq. (28) implies that $V^{(0)}$ can also be parametrized using Eq. (A.2). In Sec. III C we took advantage of this parametrization of $V^{(0)}$ in our study of the CKM renormalization.

B. Non-holomorphic part of the up-quark self-energy

The non-holomorpic part of the up-quark self-energy is not chirally enhanced and, since it therefore does not lead to large corrections to the Yukawa couplings, the CKM matrix or the fermion wave functions, it has been ommitted from eqs. (10)-(12). Note, however, that its

contribution to the effective Higgs-quark-quark couplings $\Gamma^{H^0/A^0\,LR\,\text{eff}}_{u_f u_i}$ and $\Gamma^{H^\pm\,LR\,\text{eff}}_{d_f u_i}$ receives a relative $\tan\beta$ enhancement with respect to the $\cot\beta$-suppressed tree-level coupling (see Eq. (48)). While the vertices $\Gamma^{H^0/A^0\,LR\,\text{eff}}_{u_f u_i}$, $\Gamma^{H^\pm\,LR\,\text{eff}}_{d_f u_i}$ do not play a role for most phenomenological applications because of their $\cot\beta$-suppression, they are important for the Higgs contributions to $b \to s\gamma$. In the following we thus give formulae for the gluino-, neutralino- and chargino- contributions to the non-holomorphic part of the up-quark self-energy.

The non-holomorphic part of the gluino contribution is easily obtained from Eq. (10) by inserting $\Delta^{u\,LR}_{ij} \to -v_d A'^u_{ij} - v_d\,\mu\,Y^{u_i(0)}\,\delta_{ij}$. The non-holomorphic part of the neutralino contribution (including neutralino mixing) is given by

$$\begin{aligned}
\Sigma^{\tilde\chi^0\,LR}_{u_i u_j} &= \frac{-1}{16\pi^2}\sum_{m=1}^{3}\left(-\frac{2}{9}g_1^2 M_1 \sum_{i',j',n=1}^{3}\Lambda^{u\,LL}_{m\,ij'}\Delta^{u\,LR}_{j'i'}\Lambda^{u\,RR}_{n\,i'j}B_0\left(\left|M_1^2\right|, m^2_{\tilde q^L_m}, m^2_{\tilde u^R_n}\right)\right.\\
&+ \left(\frac{g_1^2}{6}v_d M_1\mu C_0\left(\left|M_1^2\right|,|\mu|,m^2_{\tilde q^L_m}\right) - \frac{g_2^2}{2}v_d M_2\mu C_0\left(\left|M_2^2\right|,|\mu|,m^2_{\tilde q^L_m}\right)\right)\Lambda^{u\,LL}_{m\,ij}Y^{u_j}\\
&- \left.\frac{2}{3}g_1^2 v_d M_1\mu Y^{u_i}\Lambda^{u\,RR}_{m\,ij}C_0\left(\left|M_1^2\right|,|\mu|,m^2_{\tilde u^R_m}\right)\right), \qquad \text{(B.1)}
\end{aligned}$$

where again $\Delta^{u\,LR}_{ij} \to -v_d A'^u_{ij} - v_d\,\mu\,Y^{u_i(0)}\,\delta_{ij}$ must be substituted. Note that, as in the case of down-quarks, only the neutralino mixing induced by a coupling to the "wrong" Higgs gives a finite contribution while the holomorphic part which includes neutralino mixing would be divergent. The non-holomorphic part of the chargino contribution reads

$$\begin{aligned}
\Sigma^{\tilde\chi^\pm\,LR}_{u_i u_j} &= \frac{-1}{16\pi^2}\mu\sum_{m,i',j'=1}^{3}\left(\sum_{n,i'',j''=1}^{3}V^{(0)}_{i'i}Y^{d_{i'}}\Lambda^{d\,RR}_{n\,i'i''}\Delta^{\tilde d\,RL}_{i''j''}\Lambda^{d\,LL}_{m\,j''j'}V^{(0)\star}_{j'j}Y^{u_j}C_0\left(|\mu|^2, m^2_{\tilde q^L_m}, m^2_{\tilde d^R_n}\right)\right.\\
&- \left. v_d g_2^2 M_2\mu V^{(0)}_{i'i}\Lambda^{d\,LL}_{m\,i'j'}V^{(0)\star}_{j'j}Y^{u_j}C_0\left(\left|\mu^2\right|,\left|M_2^2\right|, m^2_{\tilde q^L_m}\right)\right). \qquad \text{(B.2)}
\end{aligned}$$

where one has to substitute $\Delta^{d\,LR}_{ij} \to -v_d A^d_{ij}$.

C. CKM renormalization in the case of CKM-dependent up-quark self-energies

In the case of non-degenerate left-handed squark masses, the up-quark self-energies depend on CKM elements due to the SU(2) relation between the soft mass matrices of the left-handed squarks. The up-squark mixing matrix $W^{u\,L}$ enters the gluino- and neutralino-contributions to the up-quark self-energy through $\Lambda^{u\,LL}_{m\,ij} = (W^{u\,L\star})_{im}(W^{u\,L})_{jm}$. The SU(2) relation (8) implies $\Lambda^{u\,LL}_{m\,fi} = V^{(0)}_{fj}\Lambda^{q\,LL}_{m\,jk}V^{(0)\star}_{ik}$ and leads in this way to a CKM-dependence of $\Sigma^{u\,LR}_{fi}$, which has been made explicit in Eqs. (10), (11). If we assume that the off-diagonal elements $\Lambda^{q\,LL}_{m\,fi}$ are at most of the same order in the Wolfenstein parameter λ as the corre-

sponding elements V_{fi} of the CKM matrix, we have to leading order in λ:

$$\begin{aligned}
\Lambda^{u\,LL}_{m\,12} &= \Lambda^{q\,LL}_{m\,12} + V^{(0)}_{12}V^{(0)\star}_{22}\left(\Lambda^{q\,LL}_{m\,22} - \Lambda^{q\,LL}_{m\,11}\right), \\
\Lambda^{u\,LL}_{m\,23} &= \Lambda^{q\,LL}_{m\,23} + V^{(0)}_{23}V^{(0)\star}_{33}\left(\Lambda^{q\,LL}_{m\,33} - \Lambda^{q\,LL}_{m\,22}\right), \\
\Lambda^{u\,LL}_{m\,13} &= \Lambda^{q\,LL}_{m\,13} + V^{(0)}_{13}V^{(0)\star}_{33}\left(\Lambda^{q\,LL}_{m\,33} - \Lambda^{q\,LL}_{m\,11}\right) + V^{(0)}_{12}V^{(0)\star}_{32}\left(\Lambda^{q\,LL}_{m\,22} - \Lambda^{q\,LL}_{m\,11}\right) \\
&\quad + V^{(0)}_{12}\Lambda^{q\,LL}_{m\,23}V^{(0)\star}_{33} + V^{(0)}_{11}\Lambda^{q\,LL}_{12}V^{(0)\star}_{32}. \qquad \text{(C.1)}
\end{aligned}$$

For the CKM renormalization it is important to distinguish between contributions to $\Sigma^{u\,LR}_{fi}$ which depend on $V^{(0)}_{fi}$ and those which do not. To this end we decompose σ^u_{fi} in analogy to Eq. (16) for σ^d_{fi} as

$$\sigma^u_{fi} = \widehat{\sigma}^u_{fi} + V^{(0)}_{fi}V^{(0)\star}_{ii}\varepsilon^u_{fi} \quad (f \neq i). \qquad \text{(C.2)}$$

For $i, f \neq 3$ the quantity $\widehat{\sigma}^u_{fi}$ does not depend on any off-diagonal CKM element. The parameters $\widehat{\sigma}^u_{f3}$ and $\widehat{\sigma}^u_{3i}$ depend on the CKM elements $V^{(0)}_{12}$ and $V^{(0)}_{23}$ (or equivalently on $V^{(0)}_{21}$ and $V^{(0)}_{32}$), but they neither depend on $V^{(0)}_{13}$ nor on $V^{(0)}_{31}$. The parameters ε^u_{fi} are given by

$$\begin{aligned}
\varepsilon^u_{fi} &= \frac{1}{\max\{m_{u_i}, m_{u_f}\}}\sum_{m,n=1}^{3}\left(\Lambda^{q\,LL}_{m\,ii} - \Lambda^{q\,LL}_{m\,ff}\right)\Delta^{u\,LR}_{33}\Lambda^{u\,RR}_{n\,33} \\
&\quad \times\left[\frac{2\alpha_s}{3\pi}m_{\tilde{g}}\,C_0\Big(m^2_{\tilde{g}}, m^2_{\tilde{q}^L_m}, m^2_{\tilde{u}^R_n}\Big) + \frac{g_1^2}{72\pi^2}M_1\,C_0\Big(|M_1|^2, m^2_{\tilde{q}^L_m}, m^2_{\tilde{u}^R_n}\Big)\right]. \qquad \text{(C.3)}
\end{aligned}$$

The term $\left(\Lambda^{q\,LL}_{m\,ii} - \Lambda^{q\,LL}_{m\,ff}\right)$ causes a strong GIM suppression of ε^u_{fi} culminating in $\varepsilon^u_{fi} = 0$ for degenerate squark masses. Therefore it is a good approximation to neglect higher-order effects related to ε^u_{fi} in the resummation formula for $V^{(0)}$, as it has been done in Eq. (41) using the approximation (18). For completeness we derive here an extended version of Eq. (41) resumming the effects of ε^u_{fi} to all orders.

We decompose the wave-function rotation matrix $U^{u\,L}$ as

$$U^{u\,L} = U^{u\,L}_{\rm CKM}\,\widehat{U}^{u\,L} \qquad \text{(C.4)}$$

in analogy to Eq. (33) for the down sector. The CKM-independent part $\widehat{U}^{u\,L}$ is defined by replacing $\sigma^u_{ji} \to \widehat{\sigma}^u_{ji}$ in Eq. (27), what amounts to the generalized Wolfenstein parametrization

$$\widehat{U}^{u\,L} = \widehat{U}^{u\,L}\left(\widehat{\sigma}^u_{12},\, \widehat{\sigma}^u_{23},\, \widehat{\sigma}^u_{13},\, 0\right). \qquad \text{(C.5)}$$

The CKM-dependent part $U^{u\,L}_{\rm CKM}$ is then given by

$$U^{u\,L}_{\rm CKM} = U^{u\,L}\,\widehat{U}^{u\,L\dagger} = U^{u\,L}_{\rm CKM}\left((u^{u\,L}_{\rm CKM})_{12},\, (u^{u\,L}_{\rm CKM})_{23},\, (u^{u\,L}_{\rm CKM})_{13},\, (u^{u\,L}_{\rm CKM})_{\rm Im}\right) \qquad \text{(C.6)}$$

with

$$\begin{aligned}
(u^{u\,L}_{\rm CKM})_{12} &= V^{(0)}_{12}V^{(0)\star}_{22}\varepsilon^u_{12}, \qquad (u^{u\,L}_{\rm CKM})_{23} = V^{(0)}_{23}V^{(0)\star}_{33}\varepsilon^u_{23}, \\
(u^{u\,L}_{\rm CKM})_{13} &= V^{(0)}_{13}V^{(0)\star}_{33}\varepsilon^u_{13} - V^{(0)}_{12}V^{(0)\star}_{22}\varepsilon^u_{12}\widehat{\sigma}^u_{23}, \\
(u^{u\,L}_{\rm CKM})_{\rm Im} &= -{\rm Im}\left[{\rm V}^{(0)}_{12}{\rm V}^{(0)\star}_{22}\varepsilon^{\rm u}_{12}\widehat{\sigma}^{\rm u\star}_{12}\right]. \qquad \text{(C.7)}
\end{aligned}$$

Inserting the decomposition (33) and (C.4) into Eq. (28) we obtain

$$V^{(0)} \;=\; U^{u\,L}_{\rm CKM}\,\widetilde{V}\,U^{d\,L\dagger}_{\rm CKM}\,. \tag{C.8}$$

The matrix $\widetilde{V}$ is defined in Eq. (37) and its elements are given in terms of generalized Wolfenstein parameters in Eq. (39). Solving Eq. (C.8) for $V^{(0)}$, we finally get

$$\begin{aligned}
v_{12}^{(0)} \;&=\; \frac{\widetilde{v}_{12}}{1-\varepsilon_{12}^{u}}\,, \qquad\qquad v_{23}^{(0)} = \frac{\widetilde{v}_{23}}{1-\varepsilon_{FC}^{d}-\varepsilon_{23}^{u}}\,,\\
v_{13}^{(0)} \;&=\; \frac{1}{1-\varepsilon_{FC}^{d}-\varepsilon_{13}^{u}}\left(\widetilde{v}_{13}+\frac{\widetilde{v}_{12}\varepsilon_{12}^{u}\left(\widetilde{v}_{23}-\hat{\sigma}_{23}^{u}\right)}{1-\varepsilon_{12}^{u}}+\frac{\widetilde{v}_{12}\widetilde{v}_{23}\varepsilon_{FC}^{d}}{\left(1-\varepsilon_{12}^{u}\right)\left(1-\varepsilon_{FC}^{d}-\varepsilon_{23}^{u}\right)}\right)\,,\\
v_{\rm Im}^{(0)} \;&=\; \widetilde{v}_{\rm Im}\;+\;{\rm Im}\left[\frac{\widetilde{v}_{12}\varepsilon_{12}^{u}\left(\widetilde{v}_{12}^{\star}-\widehat{\sigma}_{12}^{u\star}\right)}{1-\varepsilon_{12}^{u}}\right]\,.
\end{aligned} \tag{C.9}$$

For the application of Eq. (C.9) one has to keep in mind that $\widehat{\sigma}_{13}^{u}$ depends on $V_{12}^{(0)}$ and $V_{23}^{(0)}$. Therefore one has to proceed as follows: in a first step $v_{12}^{(0)}$, $v_{23}^{(0)}$ and $v_{\rm Im}^{(0)}$ are calculated from Eq. (C.9). The results are used to determine $\widehat{\sigma}_{13}^{u}$. With the help of $\widehat{\sigma}_{13}^{u}$ one can then calculate $\widetilde{v}_{13}$ from Eq. (39) and finally $v_{13}^{(0)}$ from Eq. (C.9).

D. Tree-level Feynman rules

The tree level Feynman rules used throughout the paper are based on those listed in Refs. [32, 33]. One should however note few differences in conventions, which we summarize in Table I. Furthermore, the bare Yukawa couplings calculated in Sec. III are in general complex, as shown explicitly in vertices displayed below. We use the convention that $Y^{f(0)}$ is the coupling appearing in the P_R component of the (pseudo-)scalar Higgs-fermion-fermion vertex, whereas $Y^{f(0)\star}$ appears in the P_L component[14].

Parameter	Current paper	Refs. [32, 33]
Down-quark and lepton Yukawa couplings	Y^{ℓ}, Y^{d}	$-Y^{\ell}, -Y^{d}$
Higgs vevs	$\langle H_{u(d)}\rangle = v_{u(d)}$	$\langle H_{u(d)}\rangle = v_{u(d)}/\sqrt{2}$
Lepton A-terms	$A^{\ell}_{ij}, A'^{\ell}_{ij}$	$-A^{\ell\star}_{ij}, A'^{\ell\star}_{ij}$
Down-squark A-terms	A^{d}_{ij}, A'^{d}_{ij}	$-A^{d\star}_{ij}, A'^{d\star}_{ij}$
Up-squark A-terms	A^{u}_{ij}, A'^{u}_{ij}	$A^{u\star}_{ij}, A'^{u\star}_{ij}$
Squark mass terms	$\Delta^{f\,LL}_{ij}$, $\Delta^{f\,RR}_{ij}$, $\Delta^{f\,LR}_{ij}$	$\Delta^{f\,LL}_{ji}$, $\Delta^{f\,RR}_{ji}$, $\Delta^{f\,LR}_{ji}$
Sfermion mass matrices	$\mathcal{M}^2_f$	$\left(\mathcal{M}^2_f\right)^{\star} = \left(\mathcal{M}^2_f\right)^{T}$

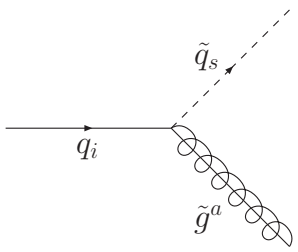
TABLE I: Differences in conventions for the MSSM parameters in the current paper and in [32, 33].

[14] Also Yukawa couplings in the LR blocks of the sfermion mass matrices, used to calculate sfermion mixing matrices, should be treated as complex. To find the correct positions of complex stars in the sfermion mass matrices, in our conventions one can use the mnemotechnic replacement rule $\mu Y \to \mu Y^{(0)}, \mu^{\star} Y \to \mu^{\star} Y^{(0)\star}$.

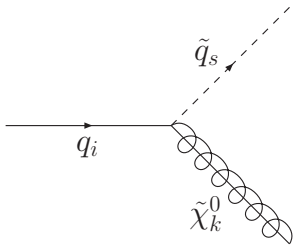
Below we list the Feynman rules for the gaugino(higgsino)-fermion-sfermion vertices. The general definitions of supersymmetric fermion and sfermion mixing matrices are given in [32, 33]. In Eq. (7) we introduced squark mixing matrices $W^{u,d}$ for the decoupling limit. These matrices can be obtained from $Z_{U,D}$ in [32, 33] substituting

$$Z_D = \begin{pmatrix} W^{dL\star} & 0 \\ 0 & W^{dR\star} \end{pmatrix} + \mathcal{O}\left(\frac{v}{M_{\rm SUSY}}\right) \quad Z_U = \begin{pmatrix} W^{uL} & 0 \\ 0 & W^{uR} \end{pmatrix} + \mathcal{O}\left(\frac{v}{M_{\rm SUSY}}\right) \quad \text{(D.1)}$$

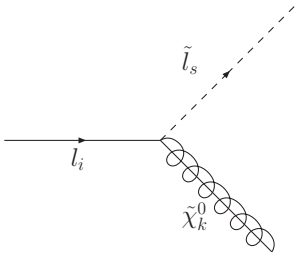
Note the complex stars on the up-squark mixing matrices in Eq. (D.1), which have been added in order to stay compatible with conventions of [32, 33].



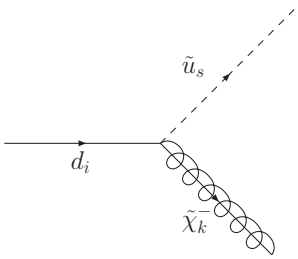
$$i\left[\Gamma^{\tilde{g}L}_{q_{i\beta}\tilde{q}_{s\alpha}} P_L + \Gamma^{\tilde{g}R}_{q_{i\beta}\tilde{q}_{s\alpha}} P_R\right] \text{ with}$$
$$\Gamma^{\tilde{g}L}_{d_{i\beta}\tilde{d}_{s\alpha}} = -g_s\sqrt{2}T^a_{\alpha\beta}Z_D^{is}$$
$$\Gamma^{\tilde{g}R}_{d_{i\beta}\tilde{d}_{s\alpha}} = g_s\sqrt{2}T^a_{\alpha\beta}Z_D^{i+3,s}$$
$$\Gamma^{\tilde{g}L}_{u_{i\beta}\tilde{u}_{s\alpha}} = -g_s\sqrt{2}T^a_{\alpha\beta}Z_U^{is\star}$$
$$\Gamma^{\tilde{g}R}_{u_{i\beta}\tilde{u}_{s\alpha}} = g_s\sqrt{2}T^a_{\alpha\beta}Z_U^{i+3,s\star}$$



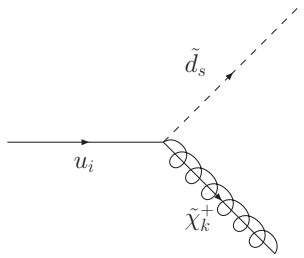
$$i\left[\Gamma^{\tilde{\chi}^0_k L}_{q_i\tilde{q}_s} P_L + \Gamma^{\tilde{\chi}^0_k R}_{q_i\tilde{q}_s} P_R\right] \text{ with}$$
$$\Gamma^{\tilde{\chi}^0_k L}_{d_i\tilde{d}_s} = \frac{1}{\sqrt{2}}Z_D^{is}(g_2 Z_N^{2k} - \frac{1}{3}g_1 Z_N^{1k}) - Y^{d_i(0)\star}Z_D^{i+3,s}Z_N^{3k}$$
$$\Gamma^{\tilde{\chi}^0_k R}_{d_i\tilde{d}_s} = -\frac{g_1\sqrt{2}}{3}Z_D^{i+3,s}Z_N^{1k\star} - Y^{d_i(0)}Z_D^{is}Z_N^{3k\star}$$
$$\Gamma^{\tilde{\chi}^0_k L}_{u_i\tilde{u}_s} = -\frac{1}{\sqrt{2}}Z_U^{is\star}(g_2 Z_N^{2k} + \frac{1}{3}g_1 Z_N^{1k}) - Y^{u_i(0)\star}Z_U^{i+3,s\star}Z_N^{4k}$$
$$\Gamma^{\tilde{\chi}^0_k R}_{u_i\tilde{u}_s} = \frac{2\sqrt{2}g_1}{3}Z_U^{i+3,s\star}Z_N^{1k\star} - Y^{u_i(0)}Z_U^{is\star}Z_N^{4k\star}$$



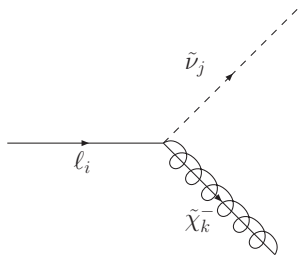
$$i\left[\Gamma^{\tilde{\chi}^0_k L}_{\ell_i\tilde{\ell}_s} P_L + \Gamma^{\tilde{\chi}^0_k R}_{\ell_i\tilde{\ell}_s} P_R\right] \text{ with}$$
$$\Gamma^{\tilde{\chi}^0_k L}_{\ell_i\tilde{\ell}_s} = \frac{1}{\sqrt{2}}Z_L^{is}(g_1 Z_N^{1k} + g_2 Z_N^{2k}) - Y^{\ell_i(0)\star}Z_L^{i+3,s}Z_N^{3k}$$
$$\Gamma^{\tilde{\chi}^0_k R}_{\ell_i\tilde{\ell}_s} = -g_1\sqrt{2}Z_L^{i+3,s}Z_N^{1k\star} - Y^{\ell_i(0)}Z_L^{is}Z_N^{3jk\star}$$



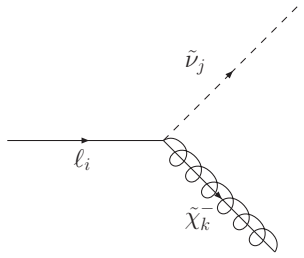
$$i\left[\Gamma^{\tilde{\chi}^\pm_k L}_{d_i\tilde{u}_s} P_L + \Gamma^{\tilde{\chi}^\pm_k R}_{d_i\tilde{u}_s} P_R\right] \text{ with}$$
$$\Gamma^{\tilde{\chi}^\pm_k L}_{d_i\tilde{u}_s} = \sum_{j=1}^{3}(-g_2 Z_U^{js\star}Z_+^{1k} + Y^{u_j(0)\star}Z_U^{(J+3)s\star}Z_+^{2k})V_{ji}^{(0)}$$
$$\Gamma^{\tilde{\chi}^\pm_k R}_{d_i\tilde{u}_s} = Y^{d_i(0)}\sum_{j=1}^{3}Z_U^{js\star}Z_-^{2k\star}V_{ji}^{(0)}$$



$$i\left[\Gamma^{\tilde{\chi}_k^\pm L}_{u_i\tilde{d}_s}P_L + \Gamma^{\tilde{\chi}_k^\pm R}_{u_i\tilde{d}_s}P_R\right] \text{ with}$$
$$\Gamma^{\tilde{\chi}_k^\pm L}_{u_i\tilde{d}_s} = \sum_{j=1}^{3}(-g_2 Z_D^{js} Z_-^{1k} + Y^{d_i(0)\star} Z_D^{j+3,s} Z_-^{2k}) V_{ij}^{(0)\star}$$
$$\Gamma^{\tilde{\chi}_k^\pm R}_{u_i\tilde{d}_s} = \sum_{j=1}^{3} Y^{u_i(0)} Z_D^{js} Z_+^{2k\star} V_{ij}^{(0)\star}$$



$$i\left[\Gamma^{\tilde{\chi}_k^\pm L}_{\ell_i\tilde{\nu}_s}P_L + \Gamma^{\tilde{\chi}_k^\pm R}_{\ell_i\tilde{\nu}_s}P_R\right] \text{ with}$$
$$\Gamma^{\tilde{\chi}_k^\pm L}_{\ell_i\tilde{\nu}_s} = -g_2 Z_+^{1k} Z_\nu^{is\star}$$
$$\Gamma^{\tilde{\chi}_k^\pm R}_{\ell_i\tilde{\nu}_s} = Y^{\ell_i(0)} Z_-^{2k\star} Z_\nu^{is\star}$$



$$i\left[\Gamma^{\tilde{\chi}_k^\pm L}_{\ell_i\tilde{\nu}_s}P_L + \Gamma^{\tilde{\chi}_k^\pm R}_{\ell_i\tilde{\nu}_s}P_R\right] \text{ with}$$
$$\Gamma^{\tilde{\chi}_k^\pm L}_{\ell_i\tilde{\nu}_s} = -g_2 Z_+^{1k} Z_\nu^{is}$$
$$\Gamma^{\tilde{\chi}_k^\pm R}_{\ell_i\tilde{\nu}_s} = Y^{\ell_i(0)} Z_-^{2k} Z_\nu^{is}$$

E. Loop integrals

The momentum dependent loop functions in Eq. (3) are defined as

$$\begin{aligned} B_0\left(p^2; m_1^2, m_2^2\right) &= \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int d^dk \frac{1}{(k^2-m_1^2)\left((k-p)^2-m_2^2\right)}\,, \\ p^\mu B_1\left(p^2; m_1^2, m_2^2\right) &= \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int d^dk \frac{k^\mu}{(k^2-m_1^2)\left((k-p)^2-m_2^2\right)}\,. \end{aligned} \tag{E.1}$$

Evaluating the function B_0 for vanishing external momentum, one gets

$$B_0\left(m_1^2, m_2^2\right) = B_0\left(0; m_1^2, m_2^2\right) = 1 + \frac{m_1^2 \ln\frac{Q^2}{m_1^2} - m_2^2 \ln\frac{Q^2}{m_2^2}}{m_1^2 - m_2^2}\,. \tag{E.2}$$

Here a divergent constant $\frac{2}{4-d} - \gamma_E + \log 4\pi$ has been dropped. It always cancels in the formulae of this article when the sum over all internal particles is performed. The same is true for the artificial scale Q^2. The loop-functions C_0 and D_0 are defined in analogy to B_0 but correspond to integrals with three and four propagators, respectively. For vanishing

external momenta they are given by

$$\begin{aligned} C_0\left(m_1^2, m_2^2, m_3^2\right) &= \frac{B_0(m_1^2, m_2^2) - B_0(m_1^2, m_3^2)}{m_2^2 - m_3^2}\,, \\ &= \frac{m_1^2 m_2^2 \ln\frac{m_1^2}{m_2^2} + m_2^2 m_3^2 \ln\frac{m_2^2}{m_3^2} + m_3^2 m_1^2 \ln\frac{m_3^2}{m_1^2}}{(m_1^2 - m_2^2)\,(m_2^2 - m_3^2)\,(m_3^2 - m_1^2)}\,, \\ D_0\left(m_1^2, m_2^2, m_3^2, m_4^2\right) &= \frac{C_0(m_1^2, m_2^2, m_3^2) - C_0(m_1^2, m_2^2, m_4^2)}{m_3^2 - m_4^2}\,. \end{aligned} \tag{E.3}$$

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